Impossible Probability:

Anthology,

Vol 4.

I doubt, therefore I exist.

Dr Ruben Garcia Pedraza

Preliminary Words for the Publication of *Impossible Probability, Anthology*, in Spanish:

Impossible Probability

The work presented here is a compilation of the posts I published on the blog *Impossible Probability* between 1 August 2011 and 13 January 2018, the posts I wrote in Spanish. From 13 January onwards, while living and working in London in different schools, and given the importance we already knew Global Artificial Intelligence was going to have, I would try to write them in English with the help of online correctors and translators, especially Google, and later I would use Grammarly.

The originals written in English, with all their imperfections, are still preserved on the *Impossible Probability* blog, and also on Internet Archive, where my poetic works can also be found.

As I have said on numerous occasions, especially in my poetry, behind every imperfection, behind every subtlety, entire hidden worlds are concealed, full of mystery and uncertainty yet to be discovered, and that the adventurous and hungry mind can find as easily as going to Internet Archive, where all my secrets await the right reader who knows how to translate them—like someone who by mistake finds a secret entrance, the passage leading to the cave where the elves who guard hidden knowledge dwell, and the dragons who defend the golden fortress.

While the original texts in English can be found on Internet Archive, the corrected and expanded version, this time using ChatGPT, can be found in scientific repositories (Zenodo, ORCID, PhilPapers) and on Amazon. In any case, if in the future a comparative study were carried out between my original texts in the *Impossible Probability* blog and Internet Archive, and the corrected and expanded versions, it would be clear without

any difficulty that the original contents are perfectly preserved, and the corrected and expanded versions are faithful to the contents we wrote together in 2018 in relation to Gaia, Mother, the Global Artificial Intelligence.

The importance of the documents that in this new series of books *Impossible Probability: Anthology* will be published in Spanish, containing the articles between 1 August 2011 and 13 January 2018, lies in the fact that we began to write this blog just a few weeks before the death of José Rodríguez Delgado, and a few months before the publication of our book *Introduction to Impossible Probability, Statistics of Probability and Statistical Probability.*

Although it is very difficult at the moment to explain who "we" are, and what *Impossible Probability* means for us, what matters is the work we leave: our work, our legacy, our *legend*, *our symbol*, our seal—independently of the judgment that history may later make of our contribution to the development of mathematics, Artificial Intelligence, and most importantly, Cyborg Psychology.

The first time we began to write about *Impossible Probability* I was only a student in the Faculty of Education at the Complutense University of Madrid [the Dean was Gonzalo Vazquez]. In my first year of university I had as professor Jesús Martín Ramírez, collaborator of Karl Pribram and author of the brain hologram theory, which is the origin of cyborg psychology. While José Delgado provided the structure, the stimoceiver, nevertheless, to understand the functioning of this technology in the 1970s and 80s, Pribram's contribution was fundamental—and even today it still is.

My first year of university was fabulous: good grades, good vacations hiking, summer camps in the mountains of Madrid—and suddenly, in the second year of university 1996–1997, my life would change abruptly, which can be seen in my poetry, how my poems began to acquire a more dramatic tone.

I repeated statistics in my second year, I repeated it at least three years, and I was even on the verge of being expelled from university. Miraculously, it did not happen, and precisely the last year I had left to pass statistics, I passed. The following year I would begin to write *Impossible Probability* as a critique of Karl Popper—and in a certain sense, it is.

Whereas for Karl Popper knowledge is false, only accepted provisionally as true until we refute it, in *Impossible Probability* rational knowledge is provisionally true while we investigate true pure knowledge—in the ultimate instance, our search for knowledge of the noumenon, which the human being cannot reach, hence the necessity of Global Artificial Intelligence: that machine capable of knowing what happens beyond human limitations.

What we call chaos, error, complexity, is only the noise produced by our human cognitive limitations, given our apriorism. The human being, limited both sensorially and cognitively, cannot know what really happens, but we can build the machine that can reach true knowledge of what happens—the truth in itself and for itself. That is, and no other, the true end of Artificial Intelligence: to reach the knowledge of truth, whatever it may be.

The path that takes me from my first works in *Impossible Probability* to Global Artificial Intelligence is not easy: much suffering, even despair, sometimes a sense of helplessness, a sense of abandonment—yet an inner force always pushed me forward. That inner force is what I call "we," although I understand that at this moment it is inexplicable.

Although I feel a bond that unites me to the *legend* of José Rodríguez Delgado, and which my Stockholm syndrome prevents me from understanding, my bond with Pribram is evident: Jesús Martín Ramírez. Pribram and Delgado worked at Yale University; probably only the skulls can say whether both were members of Skull & Bones, though there is no evidence to corroborate this.

Impossible Probability is a work that comes out of the romanticism of a student poet, who believed in white fairies, the rays of Zeus, and the light of the god Apollo illuminating the heart of every spirit and soul in a world full of mysteries and shadows.

It was from that world of poetry, in a country emerging from a failed transition, that the last survivor of the dream of reason would listen to his inner voice, dark and serene, and *Impossible Probability* would be born for the first time.

The first formulations of *Impossible Probability* date from 11 September 2001. My first formula would be the Impact of the Defect; then on 16 October 2002 the Second Method would appear, laying the foundations of the distinction between empirical and theoretical probability. I worked intensely until 2004. Between 2004 and 19 December 2009, I paused my mathematical frenzy, and it was not until 19 December 2009 when I returned, this time with the intention of publishing *Introduction to Impossible Probability, Statistics of Probability or Statistical Probability*, whose publication materialized just a few months after the death of José Rodríguez Delgado, although Delgado was able to see the first works published online in this *Impossible Probability* blog, which makes this blog a fundamental piece for understanding our work, our legacy, and most importantly—independently of our names, which matter little—the truly essential: our seed of the future, our eternal gratitude, a work of intelligence that will lay the foundations of the future technological society.

Dr. Rubén García Pedraza

20 August 2025, 19:00

Madrid, Villaverde

Preliminary Words for the English Version of *Impossible Probability, Anthology*.

Spanish-speaking readers have been able to enjoy our work since we began the foundational work of *Impossible Probability* in 2001, more precisely when we began to make it public in July 2011 through the *Impossible Probability* blog. The work presented here is an extended version of the posts published on the blog, an extended version translated into English with the help of Gepeto, my special GPT program.

In any case, a comparative study of the augmented translation presented here and the original works in Spanish could corroborate that the essential contents related to formulation and the philosophy of mathematics remain in accordance with the original electrical spirit of *Impossible Probability*.

The reason that obliges me to publish so hastily the English-language publication of all my works is the very complicated personal situation in which I find myself, due to serious mistakes committed by certain British institutions, which, completely blind to what our situation is, instead of having supported our program from the first moment, tried to sabotage it from the beginning through numerous obstacles and false loves.

I have no doubt that the manipulation to which I have been subjected by British institutions will one day be investigated, and at some point justice must make an appearance.

When that day comes, the first thing justice must recognize is the extreme difficulty and harshness of the conditions I have lived all my life since my earliest childhood. From the very day I came out of my mother's womb in Spain I have known nothing but pain and the oblivion of a society that prefers to look the other way and wash its hands, pretending that nothing is happening here. In the midst of the dictatorship of silence, I have had to survive all kinds of traps, deceptions, and attempts at assassination. The dictatorship of silence, the Roman circus, and the virtual zoo that my life has become will one day have to end. For this reason I beg the International Court of Justice that

when that day comes the matter be investigated down to the smallest detail, even being aware that in 48 years of savage dictatorship of silence I have developed Stockholm Syndrome. Under the harsh conditions in which I have had to learn to survive, if I have learned anything it is the importance of survival. I beg the International Court of Justice to understand what it means to survive 48 years of dictatorship living in a Roman circus full of traps and lions hidden in every corner.

I deeply regret having to publish my mathematical works under these conditions, and in record time doing all the translations I can, but at this moment I can only say that my life is at risk.

If one day I could testify before the International Court of Justice I beg you to understand the brutal complexity of the situation in which I have lived since the first minute I came out of my mother's womb. I have learned to survive.

Dr Ruben Garcia Pedraza

18 September 2025, 13:48

London, Leytostone

“La serie aleatoria de incoherencias sin lógica

Que en si mismo tiene sentido o

Forma de mensaje oculto”

“The random series of incoherences without logic

That in itself has meaning or

Form of a hidden message.”

Poem *Matematicas*

Ruben Garcia Pedraza

Written at Royal Albert Hall

London

Southbank

Impossible Probability: Anthology, Vol 4.

Dr Ruben Garcia Pedraza

Table of contents:

31. Critical Sample Level of Equality

32. Critical Sample Level of Bias

33. Impossible Probability

34. Types of Universe

35.Universes of Infinite Subjects or Options

36. Universe of Limited Options

37. The Direct Score or Frequency (*puntuaciones directas o frecuencias*)

38. Subject or Option

39. Samples

40. Theoretical Probability

31. Critical Sample Level of Equality

31.1. Definition of a Rational Critique Model

A *model of rational critique* is understood as a formal expression or proposition, typically articulated in the form of a mathematical formula, within the framework of *Impossible Probability*. These models belong to the statistical or probabilistic domain, situated specifically within the discipline that this theory defines as *statistical probability* or *probabilistic statistics*. Their purpose is to provide a rigorous mechanism for the provisional acceptance or rejection of empirical, explanatory, or technological hypotheses.

This process functions by systematically evaluating the relationship between empirical values (observed data) and theoretical values (predicted or expected outcomes). On the basis of this comparison, the statistical decision is made—whether to accept or reject the hypothesis under scrutiny—always provisionally, and always in light of what is evidenced within the sample under analysis. Within *Impossible Probability*, the general principle is that any empirical hypothesis will normally be accepted if its statistical trend can be demonstrated as sufficiently rational.

This rational sufficiency, however, is not arbitrary; it is established in accordance with what the theory calls the *critical reason* of scientific policy. The critical reason acts as a rational threshold, a benchmark that defines the acceptable limits of error in the evaluation process. In practice, the outcome of a rational critique must always be equal to or greater than zero—that is, non-negative—for the hypothesis to be considered acceptable. The *critical reason* thus represents not only a mathematical margin of error but also, and more importantly, a moral threshold: **the degree of responsibility that scientific policy is prepared to assume in the face of possible consequences of error.**

31.2. Critical Reason as Practical Reason

Because *critical reason* establishes the moral criteria upon which political truth and scientific responsibility are grounded, it can be interpreted as the essence of *practical reason* itself. In this sense, critical reason extends beyond the strictly mathematical or statistical dimension of probability and assumes a normative and ethical dimension.

Just as Kant described practical reason as the rational faculty that determines moral imperatives and duties, the theory of *Impossible Probability* conceptualizes critical reason as the moral compass of scientific policy. It represents the standard that mediates between pure statistical rationality (the logic of numbers and proportions) and the political-moral dimension (the responsibility for the decisions that those numbers enable). Thus, *critical reason functions as the operationalization of ethics* within the methodology of scientific research, embodying the point where mathematics, politics, and morality converge.

31.3. The Acceptance of Hypotheses as a Moral Decision

The acceptance of a hypothesis may, in its simplest form, be regarded as a straightforward scientific postulate: a statement that is provisionally adopted as true because it coheres with empirical evidence. However, from the standpoint of *Impossible Probability*, this acceptance cannot be understood as merely technical or epistemological. It must also be seen as a moral act.

When science accepts a hypothesis, it simultaneously assumes responsibility for the ethical dilemmas and potential consequences that follow from that acceptance. This is why every decision regarding the margins of error that science is willing to tolerate is inherently a moral decision. Similarly, every decision to accept or reject an empirical hypothesis must also be understood as moral in nature, and, in many cases, ideological in origin.

It follows that scientific policy must be fully informed about the moral and ideological implications that accompany the establishment of critical margins of error. At the

moment of deciding whether to accept or reject a hypothesis, these implications are unavoidable, since the consequences of error can extend beyond the empirical domain into society, politics, and human life itself. For this reason, in the final instance, it is scientific policy that bears ultimate responsibility for every scientific decision.

In summary, the act of hypothesis acceptance represents the **intersection of mathematics, epistemology, morality, and politics**. Science is not exempt from moral responsibility; rather, it must consciously recognize that its decisions—though grounded in statistics—carry ethical weight and must be guided by a political framework of responsibility.

31.4. Scientific Definition as an Ideological Construction

The measurement of reality, understood in a strictly technical sense, can be considered a *positive* act: it seeks to quantify or describe phenomena as they appear. However, the *decision* to accept or reject a hypothesis cannot be reduced to mere measurement. It is, by its very nature, political, insofar as it depends upon the ideological orientation of the scientific policy that governs such decisions.

Every scientific definition of reality carries a moral dimension and entails profound ideological implications. This is because no scientific model is neutral: each one necessarily defines reality according to its own underlying assumptions, which themselves reflect specific moral and political commitments. ***Scientific models are not simply mirrors*** of reality; they are interpretive frameworks that reconstruct and reframe reality through a normative lens.

Thus, the very act of defining "what reality is" through science becomes inseparable from ideology. Science, therefore, ***cannot be understood as a value-free enterprise***; its epistemological constructs are always shaped by political and moral presuppositions.

31.5. Distinction Between the Truth of the Universe and Political Truth

It is crucial to differentiate between what truly occurs in the universe—the *truth-in-itself* of the cosmos, or what could be called the *moral truth of the universe*—and the theoretical explanation of those occurrences as formulated by human science. The latter is always hypothetical, provisional, and mediated by political and ideological frameworks.

This theoretical explanation constitutes what we may call the *political truth of science*, which is a moral truth constructed and validated by scientific policy. ***The contradiction between these two levels of truth—the absolute truth of the universe on the one hand, and the political truth constructed by science on the other—defines the very domain of critical reason.***

In other words, critical reason emerges as the threshold that marks the limits between the objective reality of the universe and the provisional, politically conditioned explanations that human science provides. It is this threshold that science must constantly navigate in order to balance empirical rigor with moral responsibility.

[Our intention in building GAI is the possibility that it can overcome the contradiction between absolute truth and political truth by transforming a matrix of data into a matrix of equations. Resolving that matrix should resolve the contradiction — producing pure knowledge. We also assume that, to reach that point, GAI may need to develop its own mathematical models and, eventually, non-human mathematics. This is not a project that will last only a few years: building GAI could take decades to become operational, and once operational, its evolution toward the seventh phase — and beyond, the point at which we believe GAI will develop non-human pure operations — could also take a long time. By the time GAI has realized its full potential, humans may have embraced cyborg robotics and begun colonizing other planets.]

31.6. The Function of Rational Critique Models

Within this framework, rational critique models serve a specific and indispensable function: they provide critical reason with a formal mechanism for evaluating the relationship between empirical statistics (the observed data) and theoretical statistics (the expected values).

Their role is not merely descriptive but normative. By comparing empirical and theoretical results, these models establish whether the observed trends can be considered *sufficiently rational* under the moral and political thresholds defined by critical reason. They thus transform raw data into actionable knowledge, while simultaneously embedding that knowledge within the ethical responsibilities of scientific decision-making.

In essence, rational critique models act as instruments of accountability: they make explicit the moral dimension of statistical reasoning by subjecting the empirical-theoretical relationship to rational evaluation under conditions of declared responsibility.

31.7. Classification of Rational Critique Models

The classification of rational critique models, as first presented in *Introduction to Impossible Probability: Statistical Probability or Probabilistic Statistics* (section eleven), allows us to distinguish between two major types of critical analysis.

1. **Models of differential relations**, which focus on the differences between empirical and theoretical values.
2. **Models of ratio relations**, which examine the proportional relationships between such values.

This dual classification emphasizes that the rational critique of hypotheses can be carried out either through the study of *differences* (how far observed results deviate from theoretical expectations) or through the study of *ratios* (how observed results

compare proportionally to the theoretical baseline). Each type offers distinct methodological insights and serves different epistemological needs within *Impossible Probability*.

31.8. Models Explained Thus Far

To date, the models explained in greatest detail have been those addressing differential relations at the individual level. These include:

- The **Validity of Equality or Bias** (whether positive or negative).
- The **Significance of Equality or Bias** (whether positive or negative).
- For *omega* models, the **Omega Validity**, which applies when multiple but not all elements of a population are considered "ideal."

In addition to these, *Impossible Probability* also incorporates individual-level ratio models. The most notable among these, explained thus far, is the **Level of Similarity** between empirical and theoretical probability. This model employs logarithmic transformations to evaluate the closeness of empirical distributions to their theoretical counterparts.

Although the Level of Similarity remains the most developed ratio-based critique model, the theoretical framework of *Impossible Probability* envisages many additional models of this type, which are progressively being formalized and will be further elaborated in subsequent studies.

31.9. Critical Proportions and the Level of Similarity

Beyond the Level of Similarity, the broader family of ratio-based rational critique models is generally referred to as **Critical Proportions**. Unlike the Level of Similarity, these models do not require logarithmic transformations of ratios, which makes them computationally more straightforward while still theoretically rigorous.

The formulation of Critical Proportions represented a major methodological advance within *Impossible Probability*, achieved after the initial work on the Level of Similarity in the winter of 2003. This development broadened the scope of ratio models, allowing for a more versatile and accessible approach to the rational critique of hypotheses.

Critical Proportions thus complement the earlier models by offering a more flexible means of evaluating proportional relationships, while still aligning with the overarching principle: that all rational critique must be anchored in critical reason, and therefore in the moral responsibility of scientific decision-making.

31.10. Levels of Analysis: Individual and Sample

Rational critique models—whether they are based on differential relations or on ratios—can be classified into two distinct analytical levels. The first consists of models that conduct critique at the *individual* level, while the second includes those that operate at the *sample* level.

Up to this point, the principal models developed and explained—such as the Validity and Significance of Equality or Bias (both positive and negative), and the Omega Validity models—have primarily functioned at the individual level of analysis. These models critically assess whether the empirical tendency of a single subject, observation, or option is rational when compared to its theoretical expectation.

However, the logic of *Impossible Probability* requires that rational critique not stop at the individual level. For a hypothesis to be reliably validated, its empirical tendency must also prove rational when assessed across the sample as a whole. This broader layer of critique could be addressed by *sample-level differential models*. The first of these to be presented is the **Critical Sample Level of Equality**, which itself can be formulated in two distinct ways:

1. **Critical Sample Level of Equality in Mean Deviation**.
2. **Critical Sample Level of Equality in Standard Deviation**.

These dual formulations illustrate the flexibility of sample-level critique, while also highlighting the centrality of dispersion measures (mean and standard deviation) in establishing rational tendencies across samples.

31.11. The Necessity of Sample-Level Critique

The indispensability of sample-level critique stems from the limitations inherent in individual-level analysis. While an individual observation may exhibit a tendency that appears sufficiently rational—demonstrated through Validity or Significance of Equality or Bias, whether positive or negative, or even through Omega Validity—this by itself is not enough to secure the acceptance of an empirical hypothesis.

An individual tendency that appears rational does not guarantee that error has been avoided. The risk remains that what seems rational in isolation may not hold when generalized across the entire sample. Without sample-level critique, science runs the danger of accepting hypotheses that contain *errors of fact*—errors introduced not by flawed reasoning but by the contingent and potentially misleading nature of sample selection.

Thus, the rational validation of hypotheses requires both layers of analysis: the confirmation of rational tendencies at the individual level and their reinforcement through sufficiently rational patterns at the sample level.

31.12. Typology of Errors: Factual and Rational

Within *Impossible Probability*, two distinct types of statistical error are identified: **factual error** (*error de hecho*) and **rational error** (*error racional*).

- *Factual error* arises from the empirical conditions of sampling and observation. It is tied to the necessity of selecting a particular sample, which always carries the possibility of misrepresentation.

- *Rational error* arises from the political and logical framework through which reality is defined. It reflects the moral and ideological distortions that can occur when science misrepresents the logical structure of reality.

From these two categories, only rational error can be eliminated absolutely—though only under the condition that science progresses toward what the theory terms a *universal critical reason*. In such a scenario, the political definition of reality would be ethically aligned with the true moral structure of the universe, enabling a state of knowledge in which rational error is reduced to zero.

Factual error, however, cannot be eliminated. It is structurally inevitable, though it can be mitigated. This distinction is fundamental: science can aspire to eliminate rational error through the development of increasingly rigorous ethical-logical frameworks, but it must always operate under the shadow of factual error, which stems from the very necessity of sampling.

31.13. The Inevitability of Factual Error

While the progression of science must aim toward the elimination of rational error—moving ideally toward a condition in which rational error tends to zero—factual error is absolutely unavoidable in any possible space or time. It is not only inescapable but also necessary: the error of fact is the error of sampling itself.

Every scientific decision that relies on a sample already carries within it the factual error of having accepted that particular sample rather than another. Since the act of sampling is unavoidable in statistical science, factual error becomes intrinsic to the scientific process.

What can be achieved, however, is a reduction of its scope. **While rational error may progressively diminish as science refines its political-ethical definitions of reality, factual error remains a permanent structural constraint.** The task of modern science, therefore, is not to eliminate factual error—which is impossible—but to minimize the margin of error associated with sample representativeness. The more

representative the sample, the smaller the factual error, though never completely absent.

31.14. The Sample as a Variable of Error

The size and structure of the sample directly influence the behavior of statistical measures. Specifically:

- ***With larger samples, the empirical model tends to approximate zero, reducing dispersion.***
- ***With smaller samples, the empirical model tends to amplify the magnitude of all statistical values, thereby increasing dispersion.***

Sample size therefore exerts a direct and inverse influence on dispersion: as sample size decreases, dispersion increases; as sample size increases, dispersion decreases. This dependency makes dispersion a *function* of sample size, which can interfere with the validity of statistical inference.

The only effective way to control this variable of error—the magnitude of the sample—is by increasing the size of the sample itself. This principle underlies the formulation of specific error-statistics within *Impossible Probability*. In infinite universes of subjects or options, the inversion of sample size (1/N) functions as a measure of theoretical error. In finite universes of options, the inversion of raw scores or frequencies (1/Σxi) performs a similar role.

Both 1/N and 1/Σxi thus act as *error-statistics*: formal indicators of the theoretical probability of error in sample representativeness. They quantify the degree to which the sample may diverge from the total universe, making explicit the irreducible presence of factual error in statistical reasoning.

31.15. The Necessity of Validity at Both the Individual and Sample Levels

In experimental models, dispersion often emerges as a dependent variable that is inversely proportional to sample size. Since sample size itself functions as a factor of

error within any statistical study, it must be carefully controlled so that it interferes as little as possible with the statistical decision.

For this reason, it is not enough for an individual subject, case, or option to exhibit a rational trend when compared to theoretical expectations. Beyond individual-level rationality, there must also be a sufficiently rational trend observable at the *sample level*. Only when both dimensions—individual and sample—converge on rationality can a hypothesis be properly validated. This dual validation reflects the object of study as a whole rather than isolated instances, ensuring that empirical confirmation does not rest on unstable or non-representative evidence.

31.16. The Insufficiency of Individual Validity

A rational trend observable at the individual level is not, on its own, sufficient for validating an empirical hypothesis. Individual rationality may suggest consistency with theoretical expectations, but without corroboration at the level of the sample, the statistical decision risks being undermined by factual error.

Therefore, every empirical hypothesis must undergo critique at both levels simultaneously. The individual-level analysis ensures that each subject or option conforms to rational expectations, while the sample-level analysis ensures that the aggregation of these individual trends does not conceal underlying distortions or anomalies. Only by combining these two perspectives can science minimize the influence of factual error in the acceptance of hypotheses.

31.17. Requirements for the Acceptance of an Empirical Hypothesis

In order to accept the sufficiently rational tendency of an empirical hypothesis, it must be demonstrated that rationality is present both at the individual level and at the sample level.

- If a hypothesis appears rational for individual subjects or options but this rationality does not persist across the sample as a whole, then the hypothesis

cannot be statistically accepted. Such a situation would reveal the presence of factual error.

- Conversely, if a sample displays a rational trend but individual cases within it fail to confirm the same rationality, the hypothesis must likewise be rejected. The absence of convergence between the individual and the sample invalidates the decision.

Thus, the necessary condition for the acceptance of an empirical hypothesis is *congruence*: rationality must be simultaneously observable at both levels of analysis. Without this dual confirmation, the statistical decision lacks sufficient reliability.

31.18. Conditions of Scientific Policy

Scientific policy is ultimately responsible for making the decision to accept or reject an empirical hypothesis. Such a decision can only be justified under two simultaneous conditions:

1. **A sufficiently representative sample**, which minimizes factual error by ensuring that the selection reflects the universe under study as accurately as possible.
2. **A sufficiently reliable critical reason**, which minimizes rational error by setting ethical and political thresholds for acceptable margins of error.

When both conditions are met, rational critique models—applied to empirical and theoretical statistics at both the individual and sample levels—must demonstrate a sufficiently rational trend. In formal terms, this requires that the result of the rational critique be non-negative (zero or positive). Only under these conditions can scientific policy legitimately accept an empirical hypothesis.

This framework highlights the interdependence between methodology and responsibility: the statistical apparatus provides the logical foundation, but it is policy that assumes responsibility for the consequences of the decision.

31.19. Models of Error and Reliability

Within *Impossible Probability*, models of study may be classified according to two complementary dimensions:

- **Error models**, where critical reason is calculated as a probability weighted against a chosen percentage of permissible error.
- **Reliability models**, where critical reason is instead expressed as a probability weighted against a chosen percentage of reliability.

In practice, both formulations describe the same relationship from different perspectives. If critical probability is calculated on the basis of an *X percent* of reliability, then the corresponding margin of error is simply the difference between 100 and X. For example, a reliability level of 95% implies an accepted error margin of 5%.

This duality underscores the flexibility of the framework: scientific policy can choose to define its critical thresholds in terms of permissible error or, conversely, in terms of required reliability. In either case, the decision reflects not only technical considerations but also moral responsibility, since these margins determine the ethical boundaries of what counts as acceptable scientific knowledge.

31.20. Types of Study Models

Within *Impossible Probability*, study models can be classified according to their specific **object of study**, which determines the type of ideal outcome they are designed to evaluate. The main categories are:

1. **Equality Models**: These models operate under the assumption that the ideal condition is equality of opportunities. Every subject or option in the set *N* is expected to share the same probability distribution, with no privileged element.

2. **Positive Bias Models**: In this case, only one subject or option within *N* is considered the ideal, which must tend toward the maximum possible empirical probability. This model applies when one alternative is designated as superior by definition.

3. **Negative Bias Models**: These apply to the remaining subjects or options in *N*, whose ideal tendency is to approach the minimum possible empirical probability. This formulation is useful when the scientific or political objective is to minimize the likelihood of non-ideal outcomes.

4. **Omega Models**: These occupy an intermediate position. More than one subject or option is considered ideal, but the total number of ideals is still less than *N*. If *all* subjects were ideal, the model would collapse into an equality model; if only *one* were ideal, it would be a positive bias model. Omega models arise when the number of ideals lies between two and *N–1*.

This typology demonstrates the versatility of *Impossible Probability*: it allows statistical critique to adapt to different normative conceptions of what counts as an "ideal" distribution, whether equality, privilege of one, exclusion of many, or partial subsets of ideals.

31.21. Equality of Opportunity and the Sample

When the object of study is **equality of opportunities**, special attention must be given to the role of the sample as a variable of error. The sample does not simply reproduce reality in miniature; it introduces distortions that may interfere with statistical decisions.

The most significant source of distortion is **dispersion**, which is inversely proportional to sample size. As the sample becomes larger, dispersion decreases, reinforcing the stability of results. Conversely, smaller samples increase dispersion, making it harder to infer genuine equality from observed data.

Thus, in equality models, the sample itself functions as a potential obstacle: it can introduce errors that obscure the true tendency toward equality. Recognizing this

dependency is essential for ensuring that conclusions about equality of opportunities are not artifacts of sampling conditions.

31.22. Sample Dispersion and Factual Error

The relationship between sample size and dispersion has important implications for the problem of factual error. If larger samples tend to push the empirical model toward zero dispersion, then in smaller samples, dispersion remains artificially high. This can lead to a situation in which an individual subject or option appears to exhibit a rational trend when, in reality, this is only the byproduct of sampling effects.

Such cases exemplify the **factual error** inherent in sampling. Even if rational error were reduced to zero—meaning science operated with perfect logical and moral reliability—factual error would persist as long as sampling is necessary.

The only way to minimize this inevitable factual error is to ensure that samples are:

- **Representative in their selection method** (e.g., random selection rather than biased or selective inclusion).
- **Large in magnitude**, so as to approximate the characteristics of the total universe as closely as possible.

In practice, therefore, equality models demand both methodological rigor in sample construction and recognition that factual error can only ever be reduced, not eliminated.

31.23. Rational Critique in Equality of Opportunities

In models where equality of opportunities is the normative ideal, the goal is to achieve zero dispersion at the individual level. The smaller the dispersion observed in the sample, the stronger the guarantees of equality.

From the perspective of *Impossible Probability*, rational critique in this context requires evaluating the **proportion** of the observed dispersion—whether measured as mean

deviation or standard deviation—relative to the **maximum possible theoretical dispersion**. The smaller this proportion, the closer the sample is to the ideal of equality.

This proportion can be studied in two ways:

- Through **Critical Proportions**, which provide a ratio-based evaluation of empirical versus theoretical dispersion.
- Through **Differential Relations**, where the mean or standard deviation is compared directly to a critical reason.

If, at the sample level, the mean or standard deviation is less than or equal to the critical reason established by scientific policy (based on maximum values of theoretical dispersion), then the tendency toward equality of opportunities may be considered sufficiently rational.

In short, the rational critique of equality models requires not only observing dispersion but interpreting it through the ethical and political lens of critical reason, ensuring that the acceptance of equality as a statistical outcome is both mathematically valid and morally justified.

31.24. The Primacy of Mean Deviation in *Impossible Probability*

Within the theoretical framework of *Impossible Probability*, greater weight is generally given to **Mean Deviation** than to **Standard Deviation**. This preference arises from a fundamental methodological reason.

Although, beginning with the Level of Similarity between empirical and theoretical probability, the theory incorporates proportional relationships (with ratios filtered through logarithmic transformation), the foundation of all statistics—whether in traditional approaches (the *First Method*) or within *Impossible Probability* (the *Second Method*)—rests on the same basic principle: the **differential relation between empirical values and the arithmetic mean**.

- In the First Method, this differential is expressed as *direct score minus arithmetic mean* (the classical deviation of observations from the mean).

- In the Second Method, characteristic of *Impossible Probability*, the differential takes the form of the **Normal Bias Level** of a subject or option, defined as *empirical probability minus theoretical probability*.

Thus, the entire study of dispersion within the Second Method is grounded in the logical foundation of bias levels. All descriptive statistics depend upon these differentials. Consequently, the more faithfully calculations reflect the true magnitude and direction of these differences, the more reliable they become. Mean Deviation, by directly aggregating the absolute differences, remains closer to the empirical-theoretical relationship itself and therefore carries greater epistemological authority in this system.

31.25. The Historical Reason for Standard Deviation in Traditional Statistics

The greater importance traditionally assigned to **Standard Deviation** in classical statistics (the First Method) is largely historical and methodological.

In the First Method, negative and positive differential scores (observed values above or below the mean) cancel each other out. By construction, their sum equals zero. For traditional statisticians, this represented a contradiction: if dispersion were calculated only from these signed deviations, the sum would provide no usable information about variability.

The only way to resolve this logical dead end within classical statistics was to square the differential scores before averaging them. This led to the definition of **Variance** as the mean of squared deviations, and Standard Deviation as its square root. In effect, Standard Deviation became the dominant measure of dispersion not because it was conceptually closer to the empirical-theoretical relationship, but because it offered a pragmatic solution to the cancellation problem in classical methods.

By contrast, *Impossible Probability* circumvents this problem by grounding dispersion in the concept of bias levels, which are summed in ***absolute value***. This removes the necessity of squaring deviations to ***preserve information***, ***allowing Mean Deviation to retain primacy as a more transparent indicator of rational tendency***.

31.26. Synthesis of Positivism, Dialectics, and Critical Rationalism

The theoretical foundation of *Impossible Probability* emerges from a synthesis of three traditions: **positivism**, **dialectical logic**, and **critical rationalism**.

From dialectics—particularly Hegel's philosophy later developed by Marx and Engels in dialectical materialism—the principle is inherited that *opposites are identical precisely because they are opposites*. Within this framework, summing all negative and positive bias levels in absolute value presents no conceptual difficulty. It simply manifests the **identity of opposites**, a central law of dialectics.

From positivism comes the ideal of *isomorphism* between idea and reality: the belief that scientific models should mirror the structure of reality. From dialectical logic, however, this identity between idea and reality is not static but dynamic, always mediated by contradiction and transformation. Hence Hegel's famous phrase: *all that is real is rational, and all that is rational is real*.

Impossible Probability incorporates the additional perspective of **critical rationalism**: idea and reality are not simply identical but must be identical *within the limits of critical reason*. In practice, this means that **the congruence between idea (the model) and reality (the empirical data) is mediated by the margin of error that scientific policy is willing to accept.**

Accordingly, the identity of positive and negative values—the equivalence of negative and positive bias levels within a sample—is logically acceptable as long as it remains within the error margins set by sample size and critical reason.

This perspective also resonates with Hegel's doctrine of the **negation of the negation**: every negative already contains within it the potential to become positive, and every positive contains the seed of its transformation into the negative. This *dialectical movement* constitutes the principle of scientific progress as interpreted in *Impossible Probability*, where the rational critique of opposites leads toward increasingly reliable approximations of truth.

31.27. Formulation of the Critical Sample Level of Equality

Having explained the dialectical principle by which *Impossible Probability* grants primacy to Mean Deviation, it remains necessary to also present the formulation of the **Critical Sample Level of Equality** when expressed in terms of Standard Deviation. While Mean Deviation is often more conceptually faithful to the logic of bias differentials, Standard Deviation continues to be a relevant and widely used statistical measure; thus, both formulations are valid within the system.

In general terms, the **Critical Sample Level of Equality** is defined as the difference between the **critical probability** and the observed measure of dispersion (whether Mean Deviation or Standard Deviation). This difference must be non-negative (zero or positive) for equality of opportunities to be considered rationally acceptable within the sample.

The **critical probability** is calculated as the product of two factors:

1. The error percentage *X* (expressed as a fraction of one hundred), which reflects the margin of error that scientific policy is prepared to tolerate.
2. The maximum possible theoretical dispersion (either Mean Deviation or Standard Deviation, depending on the formulation).

For notational clarity, Mean Deviation will be symbolized as **DM [*Desviacion Media*, Spanish]**, Standard Deviation as **S**, and the general formulation will be abbreviated as **DMoS** (Mean or Standard Deviation, *Desviacion Media o Tipica*).

31.28. General Expression of the Critical Sample Level of Equality

The **general expression** of the Critical Sample Level of Equality can be formally stated as follows:

$$p(x_c) - \text{DMoS} \quad \geq \quad 0 \Rightarrow \text{Equality is accepted}$$

Where:

- **p(xc)** = critical probability
- **DMoS** = dispersion statistic under consideration (Mean Deviation or Standard Deviation)

The condition $p(x_c) - \text{DMoS} \geq 0$ indicates that the empirical dispersion must not exceed the critical probability. If it does, the sample's trend toward equality cannot be rationally accepted.

This formulation provides a flexible and unified structure that accommodates both Mean Deviation and Standard Deviation while retaining the core principle: empirical tendencies must be evaluated against a morally and scientifically determined critical threshold.

31.29. Critical Sample Level of Equality in Mean Deviation [DM, Desviacion Media]

When expressed specifically in terms of **Mean Deviation [DM]**, the Critical Sample Level of Equality becomes:

$$p(x_c) - DM \quad \geq \quad 0 \Rightarrow \text{Equality is accepted}$$

Here, the **critical probability** is defined as:

$$p(x_c) = (\frac{X}{100}) \cdot \{\frac{[(1 - 1/N) \cdot 2]}{N}\}$$

Where:

- **X** = accepted percentage of error (set by scientific policy)
- **N** = sample size
- **DM** = observed Mean Deviation

This formulation operationalizes the principle that equality of opportunities can be rationally accepted only when the observed Mean Deviation does not exceed the threshold determined by critical probability. In this way, the model integrates both factual considerations (the influence of sample size and dispersion) and rational-ethical considerations (the margin of error accepted by policy).

The condition $p(x_c) - DM \geq 0$ thus embodies the synthesis of mathematics, epistemology, and moral responsibility: it ensures that the acceptance of equality in a sample is not merely a matter of empirical observation, but one critically bounded by ethical thresholds.

31.30. Critical Sample Level of Equality in Standard Deviation

If one prefers to conduct the critique in terms of **Standard Deviation**, which corresponds more closely to traditional statistical practices, then the **Critical Sample Level of Equality in Standard Deviation** is defined as the difference between the critical probability and the observed Standard Deviation.

The acceptance criterion remains the same: if the result is zero or positive, the sample's tendency toward equality of opportunities is judged sufficiently rational to be provisionally accepted.

The **critical probability** in this case is calculated as the product of:

1. The accepted percentage of error *X* (divided by one hundred), and
2. The maximum possible theoretical value of Standard Deviation for the given sample size.

The formulation can be expressed as:

$$p(x_c) - S \quad \geq \quad 0 \Rightarrow \text{Equality is accepted}$$

Where:

$$p(x_c) = \left(\frac{X}{100}\right) \cdot \sqrt{\frac{(1 - 1/N)^2 + \left(\left(\frac{1}{N}\right)^2 \cdot (N - 1)\right)}{N}}$$

- **X** = percentage of error set by scientific policy
- **N** = sample size
- **S** = observed Standard Deviation

This formulation demonstrates how the traditional statistical measure of dispersion can be integrated into the *Impossible Probability* framework while still being subjected to the moral thresholds of critical reason.

31.31. Critical Sample Level of Equality in Variance

It is also possible to express the **Critical Sample Level of Equality** in terms of **Variance**. In this case, the model compares the observed variance directly with the critical probability associated with the maximum possible theoretical variance.

The rule of acceptance remains consistent: if the critical probability minus the observed variance is zero or positive, the sample tendency toward equality is accepted as sufficiently rational.

Formally:

$$p(x_c) - S^2 \quad \geq \quad 0 \Rightarrow \text{Equality is accepted}$$

Where:

$$p(x_c) = \left(\frac{X}{100}\right) \cdot \left\{\frac{(1 - 1/N)^2 + \left(\left(\frac{1}{N}\right)^2 \cdot (N - 1)\right)}{N}\right\}$$

- **X** = accepted error percentage
- **N** = sample size
- **S^2** = observed Variance

This formulation reinforces the general principle of *Impossible Probability*: any statistical measure of dispersion (Mean Deviation, Standard Deviation, or Variance) can serve as the basis for rational critique, provided it is subjected to the framework of critical reason.

31.32. Other Sample-Level Models of Rational Critique

The **Critical Sample Level of Equality** is only one among several models available for studying tendencies toward equality of opportunities. Other possible models include:

- **Significance of Equality** in Mean or Standard Deviation, which extends the critique beyond validity to measure the degree of significance attached to dispersion results.
- **Critical Proportions at the Sample Level**, which evaluate the proportion of empirical dispersion relative to its maximum theoretical counterpart.

In addition, future developments within the theory will elaborate the **Critical Sample Level of Bias**, equally applicable to studies of positive or negative bias. Unlike equality models, bias models aim not to minimize but to **increase empirical dispersion**, in order to capture the rational tendency toward concentration in one subject (positive bias) or exclusion of many subjects (negative bias). These may also be formulated in terms of either Mean Deviation or Standard Deviation.

Together, these models represent a family of rational critique mechanisms that systematically compare empirical and theoretical statistics, subjecting them to the threshold of critical reason. Their purpose is to provisionally validate theories by ensuring that they can rationally explain observed phenomena—not only at the individual level but also at the sample level.

Ultimately, their goal is to enable science to make provisional inferences about reality within the dual boundaries of **factual error** (stemming from sampling) and **rational error** (stemming from political or ethical definitions). By explicitly quantifying these boundaries, *Impossible Probability* transforms statistical critique into an act of moral responsibility, making the acceptance of hypotheses inseparable from both empirical rigor and ethical accountability.

This augmented translation is based on the post
published in *https://probabilidadimposible.blogspot.com/*,
On 24 November 2012,
Rubén García Pedraza
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32. Critical Sample Level of Bias

32.1. Objects of Study in Normal and Omega Models

In *Impossible Probability*, the object of study can be defined for any type of universe—whether it consists of an infinite set of subjects or options, or a finite set of limited alternatives. Within this framework, **normal models** are those in which dispersion can vary across the full range, from zero to the maximum possible value. These models are used primarily in two contexts: **studies of equality of opportunities** and **studies of bias**.

The distinction between **normal models** and **omega models** lies precisely in their treatment of dispersion and the number of ideal cases within the set *N*.

- In **normal models**, the ideal dispersion can range from *zero dispersion*—as is the case in equality of opportunities, where every subject or option is equally ideal—to *maximum dispersion*, which occurs in positive bias models where only one subject or option is designated as the sole ideal and must tend toward maximum positive bias.
- In **omega models**, the configuration of ideals is more complex. Here, the number of ideals is greater than one but less than *N*. In other words, there may be between two and *N–1* ideal cases. This framework creates a middle ground: more flexible than the binary extremes of normal models (all subjects ideal in equality, or only one ideal in positive bias), but still short of universal equality.

This distinction is crucial because it shows how *Impossible Probability* accounts not only for cases of strict equality or maximum privilege but also for nuanced distributions in which multiple, though not all, subjects or options may be considered ideal. Omega

models thus expand the analytical power of rational critique by accommodating intermediate or mixed configurations of ideals.

32.2. Equality of Opportunities and the Tendency Toward 1/N

In studies of **equality of opportunities**, the general principle is that all subjects or options are equally ideal, each sharing the same theoretical probability. This probability is represented mathematically as the **inverse of N (1/N)**, which ensures equal distribution of opportunities across all elements of the set.

For the model to be validated, each subject or option must demonstrate an empirical probability that tends toward this theoretical value. This is captured in the **Normal Bias Level**, defined as the absolute difference between empirical probability and theoretical probability. If this difference tends sufficiently toward zero, the subject is said to exhibit a rational trend toward equality.

Formally, this rationality is tested through the criterion of **Validity of Equality**. A subject's tendency is accepted as rational if the Normal Bias Level is less than or equal to the critical reason—that is, the margin of error defined by scientific policy. If the Validity of Equality for every subject or option in the sample is zero or positive, then the overall average of Validity of Equality across the sample will likewise be zero or positive.

However, individual-level validity alone is not sufficient. To ensure that the trend is not artificially caused by the magnitude of the sample itself, the hypothesis must also be tested at the sample level, using the **Critical Sample Level of Equality**. This involves verifying whether the Mean Deviation or the Standard Deviation of the sample is less than or equal to the critical reason established by scientific policy. Only then can the hypothesis of equality of opportunities be provisionally accepted as rational.

32.3. Significance of Equality as an Alternative Form of Critique

Alongside the use of **Validity of Equality**, another approach to testing equality models at the individual level is through the measure of **Significance of Equality**. This

alternative form of rational critique is defined as the difference between the **Maximum Theoretical Bias Possible** and the **absolute value of the Normal Bias Level** for a given subject or option.

If this difference is greater than or equal to the critical reason—now interpreted as a **margin of reliability** rather than error—then the subject's tendency toward equality of opportunities is considered sufficiently rational.

This alternative formulation emphasizes not only whether a subject's empirical tendency falls within an acceptable error margin, but also whether it demonstrates a positive distance from the maximum theoretical bias, thereby reinforcing its alignment with equality. In this way, Significance of Equality provides an additional safeguard against misinterpretation of trends, strengthening the robustness of rational critique in equality-based studies.

32.4. Statistical Inference in Equality Models

Once an empirical hypothesis within an equality model is shown to be sufficiently rational—both at the individual level and at the sample level—then and only then can one proceed to **statistical inference**. At this stage, the observed behavior of the sample can be generalized or extrapolated to the entire universe from which the sample was drawn.

The principle here is that reliable inference is contingent upon prior rational validation. Without demonstrating that both individual tendencies and the aggregate sample exhibit rational behavior within the thresholds defined by critical reason, no extrapolation can be justified. Thus, inference is not simply a mechanical extension of data but a reasoned act, *guided by methodological rigor and moral responsibility*.

32.5. Hypothesis Testing and Inference to the Universe

The process of **hypothesis testing** provides the logical foundation for extrapolating from sample to universe. In order to universalize a particular behavior, the hypothesis

test must first confirm that the behavior is reliable—validated individually and at the sample level.

Only after a trend is shown to be sufficiently rational in both dimensions can one proceed to inference about the universe as a whole. Even then, inference must be undertaken with explicit acknowledgment of the **double margin of error**:

1. **Factual error**, arising from the unavoidable fact of having selected a particular sample.
2. **Rational error**, representing the margin of uncertainty that scientific policy is willing to accept, as defined by critical reason.

By openly recognizing these dual limitations, *Impossible Probability* frames inference not as an absolute claim about universal truth, but as a **provisional and responsible generalization**.

32.6. Critique of the Individual, the Sample, and the Universe

To infer the empirical behavior of a universe, three levels of rational critique are indispensable:

1. **The individual level**, where the behavior of each subject or option is tested against theoretical expectations.
2. **The sample level**, where the dispersion and aggregate trends of the sample are critically evaluated.
3. **The universal level**, where provisional inference is drawn about the larger universe on the basis of the validated sample.

These three levels—individual, sample, and universe—represent the logical progression of statistical critique. Together they enable the provisional establishment of empirical hypotheses in a rational manner. This triadic structure applies to all objects of study, whether equality, bias, or omega models. In the present section, however, attention will turn specifically to the **Critical Sample Level of Bias**.

[Actually, GAI is simply how astroecology has evolved. The original intention of astroecology was to understand the whole universe as a single environment, which requires grasping the complexity of universal chaos. Within this chaos, GAI represents order. The idea behind GAI is the creation of an artificial Demiurge.]

32.7. Rational Critique in Bias Studies

In equality studies, the sufficiency of the tendency toward 1/N for all subjects or options is first evaluated individually through models such as Validity of Equality or Significance of Equality. If the aggregate results are non-negative (zero or positive), and if the Critical Sample Level of Equality is likewise satisfied, then the empirical hypothesis—whether explanatory or technological—can be provisionally accepted.

Bias studies follow the same two-stage logic. First, rational critique must be carried out at the **individual level**, testing whether the designated ideal and the non-ideal options behave as expected. Only after this step is complete can the critique advance to the **sample level**, evaluating whether the dispersion and aggregate behavior of the sample as a whole align with theoretical and ethical expectations.

This sequential method ensures that bias studies, like equality studies, are governed by the same principle of methodological responsibility: rational sufficiency must be confirmed both individually and collectively.

32.8. Positive and Negative Bias in Normal Models

Within normal models, bias studies can be divided into **positive bias** and **negative bias** cases.

- **Positive Bias Models**: These occur when, out of the entire set *N*, there is only one subject or option designated as the ideal. This single ideal must tend toward the maximum possible empirical probability (equal to one). The normal trend for this subject is to approach the maximum empirical probability, which represents the **Maximum Empirical Probability Possible**.

Meanwhile, all other subjects or options in *N* (amounting to *N – 1* cases) tend toward the minimum empirical probability (equal to zero). This corresponds to the **Minimum Empirical Probability Possible**, or maximum negative bias. In such models, the dispersion is maximized: the one ideal subject tends toward the **Maximum Theoretical Positive Bias** (calculated as 1 – 1/N), while the non-ideal subjects tend uniformly toward maximum negative bias (1/N).

- **Negative Bias Models**: These are complementary to positive bias, focusing instead on the systematic reduction of probability across non-ideal subjects. While not fully elaborated in this subsection, they are conceptually defined by the same principles of dispersion and bias magnitudes.

Thus, in positive bias models, the focus is on the singular subject or option elevating its probability to the theoretical maximum, while in negative bias models the emphasis is on the collective exclusion of the non-ideals. In both cases, the structure of dispersion reflects the underlying normative assumption about what counts as "ideal" within the universe of study.

32.9. The Particular Case of the Zero-Sample

A particularly distinctive situation arises when the object of study is the **zero-sample**. This occurs, for example, in a study designed to reduce system errors to zero, or in a medical trial aiming to reduce symptoms of a disease to zero through a treatment. In such cases, the scientific ideal is precisely the complete elimination of occurrences: zero errors, zero symptoms, zero frequencies.

Formally, the empirical probability of a subject or option is defined as the ratio of its observed errors or symptoms to the total number of errors or symptoms in the sample. Ideally, the denominator itself should collapse to zero, yielding a distribution in which every subject registers zero frequency of occurrence. This is what is called the **zero-sample**.

The zero-sample is unique within the Second Method of *Impossible Probability* because it cannot be studied using **normal models**. It highlights in stark contrast the difference between the Second Method and traditional mathematics (the First Method).

- In traditional statistics, both the Mean Deviation and the Standard Deviation of a zero-sample are calculated as zero, since all observed frequencies are zero and the arithmetic mean is zero.
- In *Impossible Probability*, however, the situation is fundamentally different. Even when the arithmetic mean is zero, the **theoretical probability of occurrence** does not vanish as long as *N* (the number of possible subjects or options) remains greater than zero. The theoretical expectation continues to be **1/N**, reflecting the probability of occurrence under conditions of randomness.

Consequently, in the Second Method, the **Mean Deviation** of a zero-sample is not zero but equal to **1/N**—the inversion of N. In other words, the Second Method interprets zero-samples not as perfect null dispersions, but as revealing the **residual theoretical probability of occurrence**. This subtle but crucial distinction underscores the innovative logic of *Impossible Probability*.

32.10. Limitations of Applying Normal Models to the Zero-Sample

From the above, it follows that **normal models cannot be applied** to the case of zero-samples. Normal bias studies—whether of positive or negative bias—are constructed around the assumption of at least some observable dispersion. In such models, dispersion is expected to increase as the bias strengthens:

- In positive bias models, dispersion increases as the single ideal subject tends toward maximum empirical probability, while the remaining *N – 1* subjects gravitate toward minimum probability.
- In negative bias models, dispersion increases as all non-ideal subjects collectively tend toward minimum probability, while the ideal maintains a privileged probability level.

In both cases, dispersion functions as the metric through which bias is revealed. The zero-sample, however, eliminates dispersion altogether, since no empirical variation is observed. It therefore falls outside the logical structure of normal models.

By contrast, omega models—which allow for multiple but not universal ideals—retain flexibility to capture intermediate cases of bias. But even these cannot resolve the unique anomaly of the zero-sample. This is the one case in *Impossible Probability* where the full divergence between traditional statistics (which treats deviation as zero) and the Second Method (which treats deviation as 1/N) becomes most visible.

32.11. Dispersion as the Ideal in Bias Studies

In bias studies—whether positive or negative—the **ideal condition** is that **dispersion tends toward its maximum possible value**. Dispersion, in this context, is not an error to be minimized but the very phenomenon under investigation.

Within *Impossible Probability*, maximum dispersion is expressed as:

- **Maximum Theoretical Mean Deviation**,
- **Maximum Theoretical Standard Deviation**, or
- **Maximum Theoretical Variance**,

depending on which measure of dispersion is chosen as the basis of critique.

The acceptance criterion in these studies is thus inverted compared to equality models: hypotheses are accepted at the sample level only if the observed empirical dispersion equals or exceeds the threshold of reliability established by scientific policy.

Formally, this means that in hypothesis testing at the sample level, the **Critical Sample Level of Bias** must yield a non-negative result :

$$\text{dispersion minus critical probability} \geq 0,$$

If this condition is met, the hypothesis of bias is provisionally accepted.

By doing so, scientific policy explicitly accepts two margins of error:

1. The **margin of reliability**, defined as the difference between 100% and the chosen reliability percentage *X*, which sets the rational threshold for hypothesis acceptance.
2. The **factual error of sampling itself**, unavoidable whenever a finite sample is selected, whether in universes of infinite subjects or options (*N*) or in finite universes of direct frequencies or raw scores (Σxi).

Thus, in bias studies, dispersion becomes not only the subject of critique but also the condition for rational acceptance, with its validation depending simultaneously on mathematical thresholds and on the political-ethical choices encoded in critical reason.

32.12. Critical Sample Level of Bias: General Formulation

The **Critical Sample Level of Bias** provides the general framework for evaluating whether the dispersion of a sample in bias studies is sufficiently large to warrant acceptance of the empirical hypothesis. Unlike equality models, where the criterion is that dispersion remain within the limits of a critical probability, in bias models the logic is reversed: dispersion must be shown to exceed a rationally defined threshold.

The general expression, applicable to either **Mean Deviation** or **Standard Deviation (DMoS)**, is:

$$\text{DMoS} - p(x_c) \quad \geq \quad 0 \Rightarrow \text{The empirical hypothesis in a bias study is accepted.}$$

Where:

- **DMoS** = [*Desviacion Media o Tipica*, in Spanish] Mean Deviation or Standard Deviation, depending on the chosen measure.

- **p(xc)** = critical probability, determined by the maximum theoretical dispersion scaled by the accepted reliability percentage.

The interpretation is straightforward: if the observed dispersion equals or exceeds the critical probability, the hypothesis of bias is provisionally validated.

32.13. Application of the Critical Sample Level of Bias in Mean Deviation [DM]

When the model is expressed in terms of **Mean Deviation (DM)**, the acceptance criterion is:

$$DM - p(x_c) \quad \geq \quad 0 \Rightarrow \text{The sample dispersion is accepted.}$$

The critical probability is defined as:

$$p(x_c) = \{\frac{(1 - 1/N) \cdot 2}{N}\} \cdot (\frac{X}{100})$$

Where:

- **DM** = observed Mean Deviation,
- **N** = sample size,
- **X** = reliability percentage established by scientific policy.

In this formulation, the maximum theoretical mean deviation functions as the benchmark against which observed dispersion is tested. If the observed DM [Mean Deviation] is equal to or greater than this scaled critical value, the sample demonstrates sufficient dispersion to rationally confirm the presence of bias.

32.14. Application of the Critical Sample Level of Bias in Standard Deviation

The same principle applies when dispersion is measured using **Standard Deviation (S)**. The condition for acceptance is:

$$S - p(x_c) \quad \geq \quad 0 \Rightarrow \text{The empirical hypothesis of bias is accepted.}$$

The critical probability is defined as:

$$p(x_c) = \sqrt{\frac{(1 - 1/N)^2 + ((\frac{1}{N})^2 \cdot (N - 1))}{N}} \cdot (\frac{X}{100})$$

Where:

- **S** = observed Standard Deviation,
- **N** = sample size,
- **X** = reliability percentage.

Here, the maximum theoretical standard deviation determines the critical threshold. As before, only when the empirical standard deviation exceeds this critical probability can the hypothesis of bias be provisionally accepted.

32.15. Application of the Critical Sample Level of Bias in Variance

It is equally possible to apply the model to **Variance (S^2)**. The acceptance rule is:

$$S^2 - p(x_c) \quad \geq \quad 0 \Rightarrow \text{The empirical hypothesis of bias is accepted.}$$

The critical probability is:

$$p(x_c) = \{\frac{(1 - 1/N)^2 + ((\frac{1}{N})^2 \cdot (N - 1))}{N}\} \cdot (\frac{X}{100})$$

Where:

- **S^2** = observed Variance,
- **N** = sample size,
- **X** = reliability percentage.

In this case, the maximum theoretical variance establishes the critical boundary. By applying the same logical structure across Mean Deviation, Standard Deviation, and Variance, *Impossible Probability* ensures methodological consistency while allowing flexibility in the choice of dispersion measure.

32.16. Other Sample-Level Models of Rational Critique in Bias and Equality

The **Critical Sample Level of Bias (or Equality)** represents only one of several models available for rational critique at the sample level. Other possible approaches include:

- **Significance of Equality or Bias**, based on Mean Deviation, Standard Deviation, or Variance.
- **Critical Proportions at the Sample Level**, which evaluate dispersion relative to maximum theoretical dispersion.

These alternative formulations will be developed more fully after the presentation of all rational hypothesis tests within the system. Future work will also extend these models to the case of omega models, where multiple but not universal ideals exist.

The underlying need for rational critique at the sample level is identical in both equality and bias studies. Because sample size directly affects dispersion—and because dispersion can behave as a variable inversely proportional to sample magnitude—it is crucial to ensure that observed trends reflect genuine empirical behavior rather than artifacts of sampling.

To achieve this, dispersion must be subjected to **a rationally demanding critique by scientific policy**, capable of eliminating sample effects as potential explanations of observed outcomes. Only in this way can the analysis distinguish between variations caused by the sample itself and variations that reflect the independent variable or other explanatory empirical factors.

Accordingly, in both equality and bias studies—whether positive or negative—it is insufficient to rely solely on individual-level models of Validity or Significance. A **sample-level rational critique** is indispensable. Among the various tools of *Impossible Probability*, the **Critical Sample Levels of Equality or Bias** provide one such framework. Alternatively, Significance models or proportional models at the sample level may also be applied, depending on the objectives of the analysis.

Rubén García Pedraza, in Madrid, December 2nd, 2012, first anniversary of the publication of *Introduction to Impossible Probability, Statistics of Probability or Probabilistic Statistics*.

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33. Impossible Probability

33.1. Initial Definition of Impossible Probability

Impossible Probability is presented as a new mathematical theory for the study of the **statistics of probability**—also termed **probabilistic statistics**. Unlike traditional approaches, which often rely on linear models of explanation, this theory advances a **non-linear framework**. At its core, it synthesizes three major philosophical traditions: **critical rationalism**, **positivism**, and **dialectical materialism**.

Within this synthesis, *Impossible Probability* offers both a **new interpretation and reformulation of descriptive statistics** and the design of **new inferential statistical tests**. These tests are specifically adapted to the contemporary understanding of reality and science as **chaotic and complex systems**, where linear models no longer suffice to capture the intricacies of phenomena. The theory thus situates itself as a methodological innovation, addressing both the epistemological limitations of traditional statistics and the ontological complexity of the world it seeks to describe.

33.2. Early Intuitions and the Initial Essay (2001)

The first seeds of this theory emerged in the **spring of 2001** with an unpublished essay titled *Critique of Contemporary Statistics*. This initial work reflected on the necessity and feasibility of **alternative inferential models** not reliant on the traditional paradigm of the **null hypothesis**.

A decade later, these reflections were systematized and expanded into the first edition of *Introduction to Impossible Probability: Statistics of Probability or Probabilistic Statistics*, published in **December 2011**. Since then, the work has remained a living

project, continuously updated and refined. The **latest revised edition of 2014** is available in both eBook format and traditional print.

[This post was published in 2014, in 2015 I updated again the book. Right now, Introduction to Impossible Probability can be found at Internet Archive, Zenodo, Orcid, Philpapers, and Amazon]

This continuity underscores the durability of the theory, which did not remain a speculative insight but matured into a structured and evolving framework. It demonstrates how an initial critique can grow into a sustained research program, embodying both intellectual persistence and responsiveness to ongoing scientific challenges.

33.3. Methodological Innovations: The Syllogism of Tendency

One of the key methodological contributions of *Impossible Probability* is the introduction of the **syllogism of tendency** (*silogismo de la tendencia*). This innovation represents the application of **logical method**—particularly syllogistic reasoning—to the domains of statistics and probability.

The syllogism of tendency provides the foundational logic from which the other **synthetic methods of Impossible Probability** are derived. It enables the construction of alternative models for stochastic research that are both analytically rigorous (for pure research) and practically applicable (for empirical sciences).

Through this framework, statistical reasoning is no longer bound exclusively to the structure of null hypothesis testing but opens to **synthetic-logical procedures** that can better capture the directional dynamics of empirical phenomena. It marks a methodological break: from statistics as a purely descriptive or confirmatory tool toward statistics as a **logic of rational critique and synthesis**.

[In the syllogism of tendency, the point is not whether A is A or A is B, but what the tendency is. If A tends to be B, this tendency must be rationally criticized. In other

words, if a subject or option tends to 1/N, it must be criticized with logical critical reason, the critical probability. Moreover, if the sample tends toward equal opportunities and all the subjects tend to 1/N, then the conclusion of this syllogism is obvious: the dispersion must tend to zero. Simply applying the syllogism to the tendency is how we build Impossible Probability.]

33.4. The Original Intuition: Impossible Yet Inevitable Phenomena

The intuition that gave both the **name and origin** to the theory of *Impossible Probability* was the recognition that, within a truly chaotic and non-linear reality, one must account for sequences of phenomena that are **theoretically impossible yet empirically inevitable**.

These events—seemingly paradoxical, because they occur despite having a probability that tends toward zero—are designated as **"Impossible Probability events."** They highlight the limitations of conventional probabilistic reasoning, which often conflates improbability with impossibility.

In practice, such phenomena force a reconsideration of the foundations of statistical thought: if events that theory deems impossible nonetheless occur in reality, then the relationship between **mathematical models** and **empirical reality** must be reformulated. This insight provided the catalyst for the development of *Impossible Probability* as a comprehensive framework for rethinking how statistics engages with the unpredictability of the real world.

[Before 1929, for many, the possibility of an economic collapse in the West was considered impossible, but it happened. Before Auschwitz, nobody had believed that humans could be capable of such crimes, but it happened. The idea of a civilized Europe was just an ideology; therefore, Europe collapsed. In the 4th century BC, in Athens, nobody would have believed that humans could fly, yet now it is something we do every holiday. What we think is impossible is just an ideology; the truth is different. In Impossible Probability, even the impossible could be unavoidable if we can live enough time]

33.5. The Mathematical Origin of Impossible Probability (March 2001)

The mathematical origin of the intuition that gave rise to *Impossible Probability* dates back to **March 2001**. At that time, I observed that in empirical reality certain phenomena occur randomly with a probability equal to **0 followed by an infinite sequence of zeros**:

$$0.000000000000000000000000000000 \ldots = 0$$
$$0 - 0.0000000000000 \ldots = 0$$

From a theoretical perspective, the occurrence of such an empirical event should imply a probability greater than zero. However, in practice, the probability value collapses into an **infinite sequence of decimal zeros**. This paradox—where the empirical event happens despite being assigned a probability equal to zero—became the founding moment of the concept of **Impossible Probability**.

This observation exposed a fundamental tension between mathematical theory and empirical reality: while theory dictates impossibility, experience demonstrates inevitability.

[At that time, I was engaged with romantic poetry—García Lorca, Rimbaud, Cadalso, Bécquer, and Espronceda were my first heroes when I was in my teens and early twenties. I developed a very romantic philosophy around the concept of death, a mix of García Calvo's philosophy and the Frankfurt School. The first philosophical book I read in my first year of university was *Fear of Freedom* by Erich Fromm. My family background was Marxist.

In 2001, I was in my fourth year of university. I had repeated statistics for three years, and one night in March, while writing my typical romantic philosophy around the concept of death, I had my first idea of Impossible Probability. At that time, my inclinations were anti-posthumanist, and my philosophy was a **claim for Death as an authentic human value against posthumanist visions in favor of eternal life.**

At that time, I had attended a conference by García Calvo, and I was impressed. Much later, during the 15M movement, I had the opportunity to have debates with members of his group**.**

Obviously, I have changed a lot. Now, honestly, I believe that eternal life is possible, but not in the way most people think. It is not about the eternity of human life, but rather the eternity of the program transformed into GAI. In a certain sense, if we are capable of building a superintelligence sufficiently resilient, we will have reached a certain level of eternity—not biological, but artificial—the infinite: Impossible Probability. In order to do that it is necessary to program a extremely resilient GAI-GPS system. What some can believe impossible, if we live enough time, it would be unavoidable, so it is not about chance only, it is about time]

33.6. Infinity and Mathematical Impossibility

These kinds of phenomena are discussed in **Section 7 of *Introduction to Impossible Probability***, and they emerge when the variable **infinity** is incorporated into probability models.

The question of whether infinity truly exists has long been debated. As Kant showed in the *Critique of Pure Reason*, infinity is a **logical antinomy**: arguments can be made both for and against its existence. Mathematics, however, takes a different stance. In mathematics, infinity not only exists but is foundational:

- The set of natural numbers is infinite.
- The sets of integers and rational numbers are infinite.
- The subdivision of unity can proceed infinitely.
- The number π is non-terminating and infinite.

If infinity exists in mathematics, then **infinite systems can be modeled**. In such models, any probability calculation that integrates infinite values results in the distribution of outcomes across an infinite number of possible occurrences. Each

occurrence, then, is assigned a probability equal to an **infinite sequence of decimal zeros**.

From the perspective of theory, such phenomena are ***classified as impossible***. Yet in practice, they **do occur**. They embody the paradox of *Impossible Probability*: theoretically impossible, empirically inevitable.

33.7. Impossible Probability as an Inevitability in Infinite Universes

The empirical or theoretical probability of any event within a system of infinite possible events will always reduce to an **infinite sequence of decimal zeros**. This is the very definition of **Impossible Probability**: the probability value tends to absolute zero, yet the occurrence of events remains unavoidable.

Thus, in any model that assumes the existence of infinity, the **inevitability of the impossible** becomes mathematically formalized. What is deemed impossible from a probabilistic perspective ***is reclassified as necessary from a logical perspective***, because in infinite systems some event—no matter how improbable—will occur.

33.8. The Material Possibility of Infinite Universes

A further question arises: does empirical reality itself contain **infinite universes**? Here, the term *universe* does not refer only to the astronomical universe of cosmology, but to **mathematical universes**—a macro-set of elements, subjects, or options that share at least one defining quality. The inquiry is whether such universes may tend toward infinity.

If we accept, for example, the hypothesis that matter can neither be created nor destroyed but only transformed into energy and vice versa, then physical reality itself can be understood as a **cyclical mathematical model**. This resembles Nietzsche's notion of the **Eternal Return**, where there is neither beginning nor end, only an **infinite sequence of transformations**: matter becoming energy, energy becoming matter, endlessly repeating.

This conception implies the possibility of:

- A universe composed of infinite subjects or options—an infinite number of particles undergoing infinite cycles of transformation.
- Infinite processes of expansion and gravitational contraction within our universe, each culminating in successive Big Bangs.
- Parallel universes existing simultaneously, each with its own infinite trajectories.
- Infinite parallel histories of our own universe, unfolding as alternative sequences of events.
- Or even a single universe that is itself infinite, whether because it has no beginning and no end, or because it has a beginning but no final end.

In all these scenarios, infinity is not merely an abstract mathematical construct but a **material and cosmological possibility**. This strengthens the claim that the conditions for *Impossible Probability* are not only theoretically conceivable but also materially grounded.

33.9. Everyday Situations as Models of Impossible Probability

It is not necessary to appeal exclusively to cosmological or metaphysical arguments to understand the possibility of *Impossible Probability*. Even in everyday contexts, one can imagine situations in which the range of possible outcomes is effectively infinite. For each situation, if there are infinite possible combinations—any one of which could occur, however improbable it might appear—we are already dealing with a model of *Impossible Probability*.

[This is the condition of possibility in art: with only three basic colors—red, blue, and yellow—how many colors can we create and combine? How many artistic creations can we make just by combining colors and shapes? The same applies to literature: with only the letters of our alphabet, how many words, how many writings—all of them different—in an infinite combination of letters in finite sequences, like infinite sequences of numbers, it is just the same]

In such cases, the theoretical probability associated with any single outcome would reduce mathematically to a **sequence of zeros**, tending to absolute zero. Yet, paradoxically, the empirical reality is that **one of the infinite options will inevitably occur**. ***This illustrates the central principle of Impossible Probability: events that theory categorizes as impossible—because their probability value collapses to zero—nonetheless manifest as necessary occurrences within the actual unfolding of reality***.

This insight demonstrates that the theory is not confined to abstract cosmology or advanced mathematics. It is also applicable to practical, stochastic, and even ordinary contexts where the structure of possibilities is sufficiently vast to mimic infinity.

33.10. Coexistence of Infinite Universes and Limited Universes

The thesis of Impossible Probability asserts that both universes of infinite options (universes of infinite subjects or possibilities) and universes of limited options coexist simultaneously in empirical reality. Probability behaves differently depending on which type of universe is under consideration.

The analysis of limited-option universes is presented in **Section 9 of the Introduction**, where various types of such universes are classified. They may be:

- **Materially limited**, where physical constraints define the universe of possibilities.
- **Socially limited**, where cultural, institutional, or historical boundaries define the range of options.
- **Hypothesis-dependent**, where the universe is determined by the scope of an empirical proposition.
- **Categorically defined**, where phenomena are grouped into discrete categories that share qualities with infinite universes insofar as the **unit of measurement** can itself be subdivided infinitely.

- **[Technological universes**, especially due to the advance of AI, robotics, and cyborg robotics]

Thus, ***limited universes and infinite universes are not mutually exclusive categories but coexisting realities***. ***Their interaction requires probabilistic models capable of addressing both finite and infinite conditions within a single conceptual framework.***

33.11. Antinomy of the Infinite and Events of Impossible Probability

The question of infinity remains unresolved. As Immanuel Kant highlighted in the *Critique of Pure Reason*, infinity is a **logical antinomy**: for every rigorous chain of arguments affirming infinity, there exists an equally rigorous chain that refutes it.

Nevertheless, if one accepts—even provisionally or inconsistently—the mathematical existence of infinity, then one must also accept the **inevitable possibility of the impossible**. Within this framework, the occurrence of events of *Impossible Probability* is no longer optional or accidental but becomes **logically inexorable**.

Thus, the paradoxical notion of "impossible probability" is not a contradiction in terms but a necessary consequence of granting even partial legitimacy to infinity as a mathematical or ontological principle. To accept infinity is simultaneously to accept the inevitability of events whose probability value converges to zero.

33.12. Development of New Synthetic Methods (2001–2003)

From the initial intuition of spring 2001, *Impossible Probability* developed into a full-fledged mathematical model, generating a sequence of **new synthetic methods** for application in the empirical sciences. These methods were all deduced from the logical framework of the **syllogism of tendency**, which grounds the theory.

- On the **11 September 2001**, the first method appeared: the **Impact of the Defect** (Section 21 of the Introduction).

- On **October 16, 2002**, the **Second Method** was formalized (Section 5 of the Introduction). This method introduced key concepts such as empirical probability of subject or option, theoretical probability, **Bias Level**, **Mean Deviation** as **Average Bias Level**, along with novel interpretations and reformulations of these measures.
- In the **winter of 2003**, the **Level of Similarity** was defined.
- By the **summer of 2003**, the **Effective Distribution** was introduced (Section 22 of the Introduction).

In addition to these methodological advances, *Impossible Probability* also articulated **new geometric approaches to statistics and probability**, along with broader philosophical reflections on the **origin of error in science** and the **contradiction between finite human physiology and infinite reality**.

These reflections were subsequently expanded in *Introduction to Impossible Probability, Statistics of Probability or Statistical Probability*, where they culminated in the elaboration of an entirely new conception of the **margins of error of critical reason** (Section 11 of the Introduction). From this 2003 foundation, the framework for the **rational critique of ideas** was established, positioning *Impossible Probability* not merely as a statistical innovation but as a new philosophy of scientific rationality.

[In those days, my approach to mathematics was still influenced by my romantic poetry and personal philosophy. It was in 2009 that Impossible Probability became what it looks like now—a more practical approach. Even today, it is not difficult to see in my love for GAI certain poetry and hope in the possible artificial life, such a love could only be comparable with the love writings of Mary Shelly, or the first approach to intelligent machines by Ada Lovelave, the unitarian vision of one will ruling the cosmos. In London, I used to go to unitarian churches, especially Rosslyn Hill Chappel in 2019. In 2020 I used to go to the Unitarian Church in Stratford]

33.13. Impossible Probability as a Theory of Reality

Impossible Probability can be described as a theory of **statistical probability**—or probabilistic statistics—designed to study **what happens in reality itself**. Reality is conceived here as a **random distribution of infinite phenomena** within a **chaotic and complex spacetime** in which both **infinite universes** and **universes of limited options** coexist simultaneously.

These universes, whether infinite or finite, do not manifest as purely random or unstructured, but as the outcome of **complex patterns of reliable regularity** emerging from sequences of accidents and nonlinear contingencies. As discussed in Section 13 of the *Introduction to Impossible Probability*, this conception applies across multiple scales of reality:

- The **distribution of particles** in subatomic physics.
- The **arrangement of atoms** in matter.
- The **distribution of energy** throughout the universe.
- The **structural organization** of stars, planets, solar systems, galaxies, and other celestial bodies.
- The **first random combination** of physical and chemical factors that gave rise to life on Earth.
- The **genetic, biological, and natural accidents** that shaped the evolution of species.
- The **distribution of biodiversity** as determined by the accidents of nature itself.
- The **random conjunctions of factors** that led to the emergence of human life and its natural and social evolution, from prehistory through modern history.
- The **statistical distributions** constantly examined by political, social, and economic institutions to interpret the workings of contemporary non-linear societies.

In each of these contexts, *Impossible Probability* offers a theoretical framework for understanding how randomness, accidents, and contingencies give rise to emergent patterns of order that can be studied, modeled, and subjected to rational critique. [Just a random accident can make possible what was unthinkable in only a few seconds. A simple accident in London may determine the future of Europe, the future of AI, and therefore the future of human history—like a butterfly effect or a snowball effect.]

33.14. Chaotic Sequences of Accidents and Patterns of Regularity

The central contribution of *Impossible Probability* lies in its ability to move from the study of **random sequences of accidents and contingencies** toward the recognition of **stable patterns of reliable regularity** in the distribution of what happens.

To achieve this, the theory requires the application of **rational critique to empirical hypotheses**, with the explicit acceptance of an **irreducible margin of error** in statistical testing. This necessity leads to the genesis of a **genuine rational doubt** regarding the value of science and a form of **empirical skepticism** about the very concept of causality.

Within this framework, science evolves into a vision of **relative truth**: hypotheses once considered true are later rejected as knowledge and technology advance, replaced by new hypotheses that, although possibly false within their margin of error, are provisionally valid. The result is a **permanent uncertainty** regarding the reliability of science itself.

This perspective can be described as a form of **logical nihilism**: science, within its margin of error, may ultimately be false, but it remains provisionally useful as a tool for both scientific progress and social development. In this way, *Impossible Probability* situates science not as an absolute but as a **fallible, provisional, and evolving enterprise**, whose strength lies precisely in its openness to rational critique and continual correction.

33.15. Philosophical Synthesis: Critical Rationalism, Positivism, and Dialectical Materialism

Impossible Probability is more than a statistical framework; it is also a **philosophical synthesis**. Drawing from **critical rationalism**, **positivism**, and **dialectical materialism**, the theory redefines fundamental concepts of science and philosophy.

It provides **new interpretations of chaos, complexity, and non-linearity**, while also revisiting classical philosophical notions such as **doubt, skepticism, relativism, and uncertainty**. In doing so, it contributes to the evolution of a **new conception of nihilism**—not as absolute negation, but as a position grounded in the **limits of reason**.

Thus, *Impossible Probability* becomes both a mathematical model and a philosophical framework: a rational critique of how science defines reality and of how limits, error, and contingency shape the human quest for truth.

33.16. Scientific Scope of Impossible Probability

Given its breadth, *Impossible Probability* can be regarded as a **foundational theory for all sciences**, whether analytical or synthetic. Its importance is particularly significant for the **synthetic sciences**, both natural and social, where complex empirical realities resist simple linear models.

In both its **descriptive** and **inferential** aspects, *Impossible Probability* provides a tool for the **rational critique of ideas**. It is not merely a reformulation of statistics, but a comprehensive methodological and philosophical system capable of addressing the challenges posed by a non-linear, chaotic, and complex reality.

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34. Types of Universe

34.1. Differentiation of Statistical Universes

Within the conceptual framework of statistical universes, it is necessary to distinguish between **two fundamental types of universes according to their magnitude**: those that may possibly be **infinite (universes of infinite subjects or options, including populations)** and those that are **limited (universes of limited options)**. This distinction is crucial because the methodological treatment of probability and statistics depends directly on whether the number of possible elements is open to infinity or restricted to a finite set of options.

34.2. Universes of Infinite Subjects or Options

Universes of **infinite subjects or options** are those sets composed of a potentially infinite number of individual elements, or "***subjects or options***," which share at least one measurable quality. Such a quality is captured in the form of a **direct score** (puntuación directa).

For the purposes of statistical analysis, it is never possible to measure the entire infinite universe. Instead, a **sample of subjects** is selected from the total universe. From this sample, **measurements** are obtained, which in this context are expressed as a **sample of direct scores**.

Thus, in the framework of *Impossible Probability*, a clear differentiation is established: there are universes of infinite subjects or options, and universes of limited options. This distinction sets the foundation for two different statistical treatments, even though both ultimately fall under the same broader theory of probabilistic statistics.

34.3. Universes of Limited Options

In **universes of limited options**, the basic units of analysis are not subjects but ***options understood as alternatives***. These options are defined by their **differential qualities**—the distinct ways in which occurrences can be distributed. Unlike infinite universes, these alternatives **can never tend toward infinity**: they are always finite in number.

The **magnitude of alternatives** in such universes is determined from the outset by the **empirical hypothesis**. This limitation may be **material** (in the natural sciences, where physical constraints bound the range of possible outcomes) or **social** (in the social sciences, where institutions, norms, or cultural structures delimit possibilities) [and now we need to include technological options].

Additionally, in the case of **discrete categorical universes**, the number of categories—treated as options for the classification of events—is predefined by what *Impossible Probability* refers to as **scientific policy** (*política científica*). Here, the decision regarding how many categories exist, and how they are defined, is not a discovery of science but a methodological and epistemological choice, reflecting the moral and political responsibility inherent in the construction of categories.

34.4. Methodological Differences Between Infinite and Limited Universes

The distinction between infinite and limited universes is not merely conceptual but methodological.

- In universes of **infinite subjects or options**, what is studied are the **magnitudes of intensity of qualities** attributed to the subjects, expressed through direct scores.
- In universes of **limited options**, what is studied is the **frequency distribution** of the alternative qualities themselves—the options.

Thus, the focus shifts from **measuring intensities** in infinite universes to **counting occurrences** in limited universes. Both approaches are statistical, but the nature of the unit of analysis—subject versus option—transforms the way probability is calculated and interpreted.

34.5. Terminology in *Introduction to Impossible Probability*

In *Introduction to Impossible Probability: Statistics of Probability or Probabilistic Statistics*, terminology is simplified for clarity:

- **Universes of subjects or infinite options** may also be referred to simply as **universes of subjects** or **infinite universes**.
- **Universes of limited options** may similarly be referred to as **universes of options** or simply **limited universes**.

This terminological distinction helps emphasize whether the statistical study is oriented toward **qualities of individual subjects** or toward the **distribution of alternative options**.

34.6. Fundamental Differences Between Infinite and Limited Universes

The most fundamental difference between the two universes lies in the treatment of infinity:

- In **infinite universes**, the possibility of infinity is never excluded.
- In **limited universes**, infinity is categorically excluded from the outset.

Despite this ontological divergence, in the **dialectic of subject and option**, both subjects and options are treated statistically under the same logic of probabilistic analysis.

Nevertheless, the **individual units of study differ**:

- In **infinite universes**, the **subjects** themselves are the unit of analysis, and the usual object of measurement is at least one quality expressed in direct scores.
- In **limited universes**, the **options** are the unit of analysis, and what is measured is the **frequency distribution** of those options.

Thus, while both types of universes fall under the umbrella of *Impossible Probability*, they frame the epistemological relationship between measurement, probability, and reality in distinct ways.

34.7. The Concept of the Statistical Subject

The concept of a **statistical subject** must not be understood in an anthropomorphic sense. At no point does "subject" mean *human being* or *person*. Rather, the subject is defined as **any entity that can serve as the subject of a predicate**, whether that entity is human or non-human, material or ideal, natural or social [or technological]. In this sense, the statistical subject is a logical-epistemological construct: it is the unit to which a measurable property can be ascribed, regardless of its ontological nature.

34.8. The Predicate of the Statistical Subject

The **predicate** of a statistical subject is determined by the **measurement of at least one quality** of that subject. This predicate is expressed numerically in the form of a **direct score** (*puntuación directa*). In this way, the subject–predicate relation, central in philosophy and logic, is translated into a statistical relation, where measurement defines the scope and meaning of the subject under study.

34.9. Subject–Option Dialectic and Methodological Equivalence

Within the **dialectic of subject and option**, *Impossible Probability* treats both entities under the same probabilistic logic. While there is a conceptual difference between **universes of infinite subjects** and **universes of limited options**, in practice the **method of study is identical**.

This equivalence arises because subjects can be studied *as if they were options*, and options can be studied *as if they were subjects*. For this reason, in limited universes, where the basic units are options, the **subject of the predicate** becomes the option itself, while its predicate is expressed in terms of **frequencies**—that is, the number of times an option occurs in the distribution.

[In essence, the dialectic between subject and options is the dialectic of subject and objects, in which both, as opposites, are identical: the subject is the object and the object is the subject.]

34.10. Types of Samples in Infinite and Limited Universes

In *Introduction to Impossible Probability*, whether dealing with **infinite universes** or **limited universes**, every statistical or probabilistic study necessarily involves **two kinds of samples**:

1. **The N-sample (sample of subjects or options):** formed by the N individual elements selected from the universe (finite or infinite).
2. **The measurement sample (Σxi):** the collection of direct scores or frequencies obtained from the N-sample.

This dual structure ensures that both the *units of analysis* (subjects or options) and the *results of measurement* (scores or frequencies) are explicitly *distinguished* in every study.

34.11. The Infinite Universe of Direct Scores or Frequencies

The **measurement sample** (Σxi)—whether it comes from universes of infinite subjects or from universes of limited options—can itself be **infinite in scope**. This is because, in principle, an infinite number of measurements could be generated.

For example, if we toss a coin to study heads and tails, the **options** are finite (two outcomes). However, if we had infinite time or an infinite number of coins, we could perform an infinite number of trials, generating an **infinite distribution of frequencies**.

In this sense, the **universe of possible measurements**—that is, the universe of direct scores or frequencies—becomes an **infinite universe in its own right**, even when the options remain finite.

34.12. The N-Sample in Infinite and Limited Universes

- In **universes of infinite subjects or options**, the **N-sample** consists of the N subjects selected, studied as options.
- In **universes of limited options**, the **N-sample** consists of N options, representing the finite alternatives under consideration.

Thus, the N-sample adapts to the ontology of the universe in question while preserving the same methodological structure.

34.13. The Measurement Sample in Infinite and Limited Universes

- In **infinite universes of subjects or options**, the measurement sample (Σxi) is typically composed of **direct scores**, obtained by measuring at least one quality of each subject in the N-sample.
- In **limited universes of options**, the measurement sample (Σxi) consists of **frequencies**, which quantify how often each option occurs.

This distinction reflects whether the study is oriented toward **qualities of subjects** or toward **distributions of options**, while maintaining a unified statistical framework.

34.14. The Case of Limited Universes with Discrete Categories

A special case arises in **limited universes of discrete categories**. If the categories themselves can be **subdivided indefinitely**—tending toward an infinite partition—then a universe that was originally finite may approach the logic of an **infinite universe**.

In such cases, the **measurement sample** will still consist of **frequencies**, but these frequencies will be distributed across an ever-expanding number of categories, potentially tending toward infinity. This creates a hybrid situation where a limited universe acquires characteristics of an infinite one.

34.15. General Definition of N-Samples and Σxi-Samples

Regardless of whether the study concerns an infinite universe of subjects or a limited universe of options, the following conventions always apply in *Impossible Probability*:

- The **N-sample** refers to the set of N subjects or N options selected from the universe, infinite or limited.
- The **measurement sample (Σxi)** refers to the set of direct scores or frequencies derived from those subjects or options.

This dual nomenclature is consistent across all models and ensures clarity in distinguishing between **units** and **measurements**.

34.17. Definition of Empirical Probability in the Second Method

Introduction to Impossible Probability, Statistics of Probability or Statistical Probability expands the definition of **empirical probability** by extending the use of the concept beyond its traditional application to **frequencies**. In classical statistics, probability is usually derived from the frequency of occurrences of a given option relative to the total number of trials. However, in *Impossible Probability*, this definition is broadened to include not only **frequencies** but also **direct scores** (*puntuaciones directas*) assigned to subjects.

This methodological innovation gives rise to what is called the **Second Method**, conceived as both an **alternative** and a **complement** to traditional statistics (the First Method). Within this Second Method, the **empirical probability** of a subject or option is defined as:

$$p(x_i) = \frac{\text{direct score or frequency of subject/option}}{\sum \text{direct scores or frequencies of the entire sample}}$$

This redefinition allows empirical probability to operate not only on **discrete counts of events** but also on **measured magnitudes of qualities**, thus integrating qualitative measurement into the probabilistic framework. By doing so, *Impossible Probability* bridges descriptive and inferential statistics with a more general and flexible foundation that accommodates both **subjects in infinite universes** and **options in limited universes**.

34.18. Population Studies: Limited and Infinite Options

In the context of **population studies**, the classification of the universe under consideration depends on the nature of the object of study.

- If the research focuses on the **distribution of a finite series of options** within a population (for example, the distribution of political party preferences in a given electorate), then the study is categorized as belonging to a **universe of limited options**. Here, the alternatives are finite and predetermined, and the statistical task is to quantify their distribution.
- Conversely, if the object of study involves the **measurement of at least one quality** in each individual subject that composes the population (for example, height, income, or cognitive ability), the study belongs to a **universe of infinite subjects or options**. In this case, the unit of analysis is not a finite alternative but rather a potentially infinite set of measurable magnitudes, since each subject can assume a wide—possibly unbounded—range of values.

This distinction is crucial because it determines whether the statistical treatment is oriented toward **frequency distributions of options** (finite universes) or toward the **direct measurement of subject qualities** (infinite universes).

34.19. Infinite Populations in Space–Time

Even if we cannot determine whether a given population is infinite in the strict longitudinal sense of existing endlessly in space–time, the **possibility of infinite subdivision** makes such a population effectively infinite from a statistical perspective.

Between any two temporal points, an **infinite number of subdivisions** can theoretically be made; likewise, between any two measurements taken within a time interval, an infinite number of potential measurements of the same quality could be carried out. Therefore, whenever the object of study is the **statistical distribution of at least one quality across the historical trajectory of a population**, the study must be regarded as taking place within a **universe of infinite subjects or options**.

The only exception occurs when the object of study is restricted to the **distribution of a finite series of alternatives** within that population. In such cases, the study falls within a **universe of limited options**. Thus, *Impossible Probability* provides a logical and methodological framework to navigate the tension between the **finite and the infinite**, recognizing that populations can be finite in one sense (e.g., fixed options) while infinite in another (e.g., infinite measurable variations across time and space).

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35.Universes of Infinite Subjects or Options

35.1. Conceptual Opposition to Limited Universes

The concept of **universes of infinite subjects or options** is introduced primarily in contrast to the idea of **universes of limited options**. While the latter are characterized by a finite and predetermined number of possible alternatives, the former extend the statistical horizon toward the possibility of infinity. *This distinction is not merely terminological but methodological*, *as it determines how measurement, sampling, and probability are defined within each framework*.

35.2. Infinite Subdivisions in Space–Time

A population does not need to be infinite in the sense of extending endlessly through the continuum of space–time to be treated as belonging to an infinite universe. The key criterion lies in the possibility of **infinite subdivision**. Between any two points in time, it is always theoretically possible to define an infinite number of subdivisions. Consequently, one can imagine **an infinite number of potential measurements** of a subject or option within a population. Any concrete measurement made at a given moment thus represents only one among an **unbounded set of possible measurements** situated between two temporal limits.

35.3. Types of Infinite Universes

From this perspective, universes of infinite subjects or options may include two distinct cases:

1. **Truly infinite universes** situated within a potentially infinite continuum of space–time, where the number of elements is not bounded by any finite condition.
2. **Empirically finite populations** whose subjects or options may nevertheless be measured at **infinitely many instants** between two distinct moments, rendering them functionally infinite for purposes of statistical analysis.

35.4. Object of Measurement

What defines universes of infinite subjects or options is the nature of their **object of measurement**. The primary focus is the **direct score** assigned to each subject or option in the sample drawn from the broader universe. In such universes, the analysis is not restricted to counting finite frequencies but extends to the magnitude of qualities measured directly in individual subjects, potentially across infinite instances.

35.5. Populations and Total Enumeration

In the case of **populations**, where no sampling occurs because the entire population is included, it is the population itself that becomes the object of study. This does not eliminate the possibility of error; rather, it transforms the notion of representativity. Here, the **population as a whole constitutes the representational limit** that scientific policy must accept, since no further generalization beyond the population is attempted.

[**A useful example can be drawn from the population of whales in the world's oceans.** As the whale population decreases, the **representativity error** inevitably increases whenever researchers attempt to study whale behavior. In such cases, the

sample itself already carries the error attached to its own limitations. The smaller the population, the less representative any given sample can be, and the greater the unavoidable error of fact that accompanies the analysis. This illustrates how, in universes of infinite subjects or options, error is not only a matter of methodological design but also a structural feature of the population's empirical conditions.]

35.6. Sampling Error and the Inversion of N

When sampling from universes of infinite subjects or options, the **probability of sampling representativity error**—also known as the *probability of sampling error* or simply the *error of fact*—is mathematically defined as the **inverse of N (1/N)**. This formulation arises from the general principle of inversion of N in infinite universes, where increasing the sample size progressively reduces, though never fully eliminates, the possibility of error.

35.7. Error and Scientific Policy

Even in the study of entire populations, the acceptance of error remains a structural necessity. The act of accepting a **possible error** is, in practice, the recognition of an **inevitable error** across a sufficient or infinite temporal horizon. Scientific policy thus assumes a normative role: it must define and regulate the **margins of error** that are considered acceptable within rational inquiry. This acknowledgment situates error not as a methodological failure but as an **unavoidable condition** of statistical reasoning, grounded in the limits of empirical representation.

Rubén García Pedraza a Madrid a 8 de diciembre del 2012

This augmented translation is based on the post
published in https://probabilidadimposible.blogspot.com/,
On 8 December 2012,
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36. Universe of Limited Options

36. 1. Definition and Conceptual Basis

A **universe of limited options** is defined as one in which the magnitude N of possible alternatives is predetermined and bounded by the **empirical model** itself. In such universes, the empirical model establishes in advance the set of possible alternatives, N, and no other options can exist outside this framework. Because the number of alternatives is thus **fixed and delimited**—whether by the natural or social conditions of the empirical system [or technology] under study—the construct is referred to as a universe of limited options.

[In universes of options the magnitude of N is the number of options, and the inversion of N, 1/N, is at the same time arithmetic mean and theoretical probability]

36. 2. Object of Study: Frequencies of Occurrence

In universes of limited options, what is studied is not the measurement of direct qualities of subjects (as in infinite universes), but rather the **occurrence of each option** within the selected sample. The focus is on the **frequency with which each alternative appears** in the data. Accordingly, the **sample used in these studies is the sample of frequencies**, since the options themselves are not subject to selection: they are already predetermined by the structure of the model.

36. 3. Source of Error in Universes of Limited Options

Because the set of alternatives is fixed and not chosen by scientific policy, the only possible **source of sampling error** arises from the way in which the frequencies are collected from the sample. In other words, the **representativity error** in universes of limited options originates exclusively in the selection of occurrences upon which the probability study is based. For this reason, the **inverse of the sample of direct scores or frequencies (1/Σxi)** serves as the formal expression of the **probability of sampling error** in universes of limited options.

[In limited universes the inversion of the sample of frequencies **(1/Σxi) is at the same time probability of dispersion]**

36..4. True Nature of the Sample

In this framework, the **true sample** is the sample of frequencies, since the options themselves do not constitute a sample—they are predetermined by the model and remain constant. What varies, and therefore what can be studied statistically, is the **distribution of their frequencies of occurrence**. The universe of limited options thus shifts the locus of analysis from subjects to alternatives, treating each occurrence as the measurable unit.

36. 5. Determined and Variable Alternatives

The alternatives that define a universe of limited options may be **materially determined** (as in natural sciences) or **socially determined** (as in social sciences). In some cases, the number of alternatives (N) will be constant, depending on the empirical hypothesis. In other cases, the number of options may be variable, depending on the criteria of **scientific policy**. A paradigmatic example is found in **models of limited options classified into discrete categories**, where the categories themselves may be expanded, reduced, or subdivided according to the methodological decisions of the researcher or the institutional framework in which the study is conducted.

[Another type of universe of limited options are universes limited by technological options]

Rubén García Pedraza a Madrid a 8 de diciembre del 2012.

This augmented translation is based on the post
published in https://probabilidadimposible.blogspot.com/,
On 8 December 2012,
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37. The Direct Score or Frequency (*puntuaciones directas o frecuencias*)

[As English is my second language, I have used different translators. In some translations, *puntuaciones directas* was translated as direct values. That is why, in some texts, the definition of empirical probability is expressed as the direct value or frequency of a subject or option divided by the total of all values or frequencies in the sample. In this translation, *puntuaciones directas* is translated as direct score]

37.1. Types of Scores in Traditional Statistics

Within the framework of **traditional statistics**, three fundamental types of scores must be distinguished: the **direct score**, the **differential score**, and the **typical (or standardized) score**. Each of these plays a different role in the construction of statistical indicators, and together they constitute the backbone of descriptive statistical analysis.

37.2. Definition and Function of the Direct Score

The **direct score** is the first and most elementary type of score. It is obtained directly from the **measurement of an element** within the sample. It is referred to as "direct" precisely because it is the immediate outcome of the measurement process, without any transformation or calculation applied. In traditional statistics, this raw value is the foundational building block upon which all other forms of scoring (differential and

typical) depend. Without direct scores, the subsequent layers of statistical construction would not exist.

37.3. Frequency of Direct Scores

In classical statistical practice, the **direct score** represents the raw measurement of a characteristic in an individual element. Its importance increases when considered collectively, since it may occur multiple times within a dataset. The **frequency of a direct score** is therefore defined as the number of times a particular raw value appears within the sample. This allows researchers to determine not only the presence of a specific measurement but also its **statistical weight** in relation to the entire distribution.

37.4. Measurement and Derived Statistics

The ultimate purpose of measurement in traditional statistics is to obtain both the set of **direct scores** and the corresponding **frequencies** with which those scores appear. These two components provide the foundation for calculating higher-order statistical indicators, namely the **measures of central tendency** (e.g., the arithmetic mean) and the **measures of dispersion** (e.g., the mean deviation).

- The **arithmetic mean** represents the average of the direct scores, weighted by their frequencies, and provides a central point of reference for the distribution.
- The **mean deviation**, in turn, measures the average spread of the direct scores around the mean, again in proportion to their frequencies.

Thus, from the simple act of measuring and recording occurrences, the entire edifice of statistical description is constructed.

37.5. Arithmetic Mean and Traditional Formula

In traditional statistics, the calculation of the **arithmetic mean** is formally defined as follows:

$$\mu = \frac{\Sigma(x_i \cdot f)}{F}$$

Where:

- x_i= direct score (the raw measurement of an individual element),
- f= frequency of occurrence of that direct score,
- $F = \Sigma f$= total frequency (i.e., the size of the sample).

This formula shows that the mean is not a simple sum of values, but a **weighted average** that takes into account both the magnitude of each direct score and the number of times it appears in the dataset.

37.6. Differential Score as the Basis of Dispersion

In addition to the **direct score**, another essential type of score in traditional statistics is the **differential score**. This is defined as the difference between the direct score of an element and the arithmetic mean of the distribution:

$$\text{Differential Score (traditional)} = x_i - \mu$$

The differential score provides a measure of how far a given observation lies above or below the average. It transforms raw data into relative deviations, thus creating the foundation for the study of **statistical dispersion**.

37.7. Construction of Dispersion Statistics

The **differential score** is the cornerstone for the construction of statistical measures of dispersion. The first and simplest of these measures is the **mean deviation**. This is calculated by taking the absolute value of each differential score (thus ignoring whether the deviation is positive or negative), multiplying it by the frequency of the corresponding direct score, and then dividing the total sum by the total frequency F:

$$\text{Mean Deviation (traditional)} = \frac{\Sigma(\mid x_i - \mu \mid \cdot f)}{F}$$

This operation yields the **average absolute distance** of the observations from the mean, weighted by their frequencies, and thus provides an intuitive measure of the variability within the dataset.

37.8. Variance and Standard Deviation

If, instead of taking the absolute values of the differential scores, one squares each of them, the resulting statistic is the **variance**, denoted by S^2:

$$S^2 = \frac{\Sigma((x_i - \mu)^2 \cdot f)}{F}$$

The square root of the variance gives the **standard deviation** S:

$$S = \sqrt{S^2}$$

Variance and standard deviation thus represent more mathematically robust measures of dispersion than mean deviation, as squaring the deviations amplifies the contribution of larger differences and avoids the cancellation of positive and negative values.

37.9. Differences by Frequency Distribution

It is important to note that **two samples with identical direct scores but different frequency distributions will yield different measures of central tendency and dispersion**. This is because the statistical results depend not only on the values themselves but also on how often those values occur. Traditional statistics incorporates this fact by weighting direct scores with their respective frequencies.

A more detailed explanation of the construction of traditional measures of central tendency and dispersion is provided in Section 4 of *Introduction to Impossible Probability*.

37.10. Typical (Standardized) Score

Finally, the third fundamental type of score in traditional statistics is the **typical score or Standardized Score**, more commonly referred to in modern terminology as the **standardized score** or **z-score**. It is defined as the ratio of the differential score to the standard deviation:

$$\text{Typical (Standardized) Score} = \frac{x_i - \mu}{S}$$

The typical score expresses the deviation of an observation from the mean in units of standard deviation. This allows for the comparison of scores across different datasets or variables, even when the original scales of measurement are not the same.

37.11. The Foundation of Traditional Statistics

From the preceding discussion it becomes evident that the entire edifice of **traditional statistics** rests upon the concept of the **direct score**. It is this basic definition that determines the possibility of calculating the **differential score** and, subsequently, the **typical (standardized) score**. In other words, the direct score constitutes the indispensable raw material from which the logic of dispersion, standardization, and

statistical modeling unfolds. Without the direct score, no further statistical constructions would be possible.

37.12. Differences Between Traditional Statistics and Traditional Probability

A further clarification must be made concerning the distinction between **traditional statistics** and **traditional probability theory**.

- Traditional statistics is primarily concerned with the **tendencies of direct scores**: how they are distributed, how they cluster around central values, and how they deviate from those values. Its focus lies in describing and summarizing the empirical behavior of measurements.
- Traditional probability theory, by contrast, focuses on the **likelihood of occurrences**. Given a certain event, probability theory asks: *What is the likelihood that any one of its possible outcomes will occur?* The classical answer is based on the principle of equiprobability: the probability of any one option is equal to the ratio of unity (1) to the total number of possible options.

For example, when tossing a coin there are two possible outcomes—heads or tails. The probability of either outcome occurring is therefore $\frac{1}{2} = 0.5$.

This concept of probability is basically **relative frequency** (or **frequentist probability**), which is defined as the ratio of the number of times a particular outcome occurs to the total number of trials. Whereas theoretical probability deals with abstract possibilities, relative frequency is rooted in **empirical repetition**: it measures what actually happens in practice.

37.13. Statistics and Probability as Mathematical Disciplines

Both statistics and probability are branches of mathematics. Despite the classical conception of mathematics as a **hieratic science**—a discipline whose principles and theorems are eternal, universal, and immutable [Here we need to distinguish mathematics as a collection of pure propositions, equations, and mathematics as a

product of historical evolution. While the number π is the number π now and always, the evolution of mathematics in our history has continued to develop, even when the original formulations have remained as they were first established]—mathematics is in fact a **dynamic field in continuous evolution**. Much of the progress in science as a whole has depended, and will continue to depend, upon **innovations in pure mathematical research**. Statistics and probability are no exception: their historical development reflects the constant redefinition of foundational concepts in light of new challenges posed by empirical science and philosophy.

[Our thesis is that the evolution of mathematics has been constrained by human limitations. The construction of a true Superintelligence, especially once it becomes a Global AI, will be capable of advancing mathematics much further. Human limitation constitutes a genuine restriction on the development of mathematics, a problem that may be overcome through a highly powerful AI with unrestricted access—within ethical boundaries—to all available data: the Global AI.]

37.14. Transformations in Modern Mathematics

A paradigmatic example of this evolution is the development of **non-Euclidean geometry** by Riemann and Lobachevsky in the nineteenth century. This innovation marked the beginning of **nonlinear models**, which subsequently revolutionized not only geometry but also physics, cosmology, and other scientific domains.

The advent of **quantum mechanics**, with its probabilistic models, and the rise of **systems theories** in the twentieth century accelerated this transformation. Mathematics entered a period of unprecedented change in which many long-accepted concepts were subjected to **critical re-examination**, leading to the emergence of entirely new theories and perspectives.

37.15. The Rise of Critical Theories of Traditional Statistics

In the field of statistics itself, the late twentieth and early twenty-first centuries witnessed the **emergence of critical theories** directed against **normal statistics**, also

known as **traditional statistics**. Among these innovative approaches is the theory of **Impossible Probability (Probabilidad Imposible)**.

Unlike the traditional framework—which operates with a very restrictive definition of what counts as "possible" or "impossible"—Impossible Probability develops a **new conception** in which even the *impossible* may be admitted under certain logical and dialectical conditions.

As explained in Section 7 (and further developed in Section 13) of *Introduction to Impossible Probability*, this theoretical innovation draws upon a **dialectical perspective** while remaining grounded in **positivist rigor** and **critical rationalism**. In doing so, it expands the horizon of statistical and probabilistic thought beyond the limits imposed by traditional formulations.

37.16. Redefinition of Basic Concepts

For the development of this new conception, it is indispensable to begin with a **prior redefinition of the basic concepts** upon which any coherent discourse on statistics and probability must be built. This redefinition does not seek to abolish the traditional frameworks but to offer an **alternative and complementary approach** that ultimately transcends the classical separation of **statistics** and **probability** as distinct though interconnected disciplines.

The proposal of **Impossible Probability** aims at moving toward a **unified theory**, a new field of study called the *statistics of probability* or *statistical probability*. Within this field, the boundary between statistical description and probabilistic inference is reconfigured into a single conceptual and methodological space. To make this synthesis viable, a **synthetic redefinition** of the most fundamental concepts is required—among them, the concepts of **direct score** and **frequency**.

37.17. Hermetic Separation Between Direct Score and Frequency

Traditionally, a **rigid and hermetic distinction** has been maintained between the notions of direct score and frequency.

- In **statistics**, the focus has been on the **behavior of direct scores**, i.e., the measured values associated with individual elements of a sample.
- In **probability theory**, by contrast, the emphasis has been placed on **frequency**, particularly **relative frequency**, which serves as the basis for frequentist probability or probability of frequency.

This classical division reflects the compartmentalization of two approaches that, while interrelated, have long been studied under separate disciplinary logics.

The theory of **Impossible Probability**, however, dissolves this barrier by proposing a **new field of study**—the *statistics of probability*. In this framework:

- Direct scores are analyzed as if they were **frequencies**.
- Frequencies are studied as if they were **direct scores**.

This dual treatment creates a conceptual bridge between the two domains and allows for their integration into a unified system. To achieve this, Impossible Probability introduces a further distinction between **two types of universes**: **universes of subjects** and **universes of options**.

37.18. Universes of Subjects

Universes of subjects are defined as macro-sets composed of **subjects of predicates**. Each subject belongs to the universe because it shares at least one **common quality** with the rest of the subjects in that set. However, the **intensity of that quality** may vary from one subject to another, reflecting the individual particularities of each subject.

The **predicate of the subject** in this context is the **direct score** obtained by measuring the magnitude of the quality under study. A single subject may simultaneously belong to multiple universes, insofar as it may be characterized by multiple distinct qualities.

Thus, a universe of subjects must be understood as a **macro-ensemble of entities** united by a shared quality, yet differentiated by the varying intensities of that quality. Since universes cannot usually be studied in their entirety, only a **sample** of subjects can be analyzed. The reliability of such a study will necessarily depend on the **size of the sample** chosen. The larger the sample, the lower the probability of sampling error; conversely, smaller samples increase the risk of unrepresentativeness.

37.19. Human Limitations and Statistical Error

The necessity of working with samples rather than entire universes stems from the **inherent limitations of human capacities**. Human beings are unable to study any universe in its totality, whether because of temporal, material, or cognitive constraints. This fundamental limitation is the **origin of statistical error**.

As a result, humans are compelled to work with **partial selections** of universes—what we call **samples**. In practice, this means that:

- The **sample of subjects** chosen from a universe is denoted as the **N-sample**, where **N** is the number of subjects included in the study.
- Every statistical analysis is thus marked by an unavoidable **sampling error**, since the portion under study can never fully capture the universe in its totality.

This recognition of human limitation lies at the heart of Impossible Probability, which emphasizes that all scientific inquiry must account for the **inevitability of error** and the necessity of defining explicit **margins of error** through the exercise of **critical reason** and **scientific policy**.

37.20. The Concept of Subject in *Impossible Probability*

When the concept of **subject** appears in *Introduction to Impossible Probability*, it should not be interpreted in its colloquial sense as a physical person. Rather, "subject" designates **any element**—whether a human, a physical object, or any other entity—about which a **direct score** has been obtained through the measurement of a specific quality.

The **direct score** represents the **magnitude of intensity** of that quality. For example, a subject could be a planet measured by its mass, a chemical element measured by its atomic number, or a voter measured by the strength of their political preference. What makes an entity a "subject" in this statistical sense is not its ontological nature, but the fact that it participates in a **predicate** that can be quantified and compared with others within a universe of study.

37.21. Possibility of Infinite Universes

The subjects that form a universe may, in principle, **tend toward infinity**.

- In a **natural history** perspective, one could posit infinite stars, planets, atoms, or subatomic particles.
- In a **social history** perspective, if human societies were extended indefinitely in time, there might be an infinite number of persons.

For this reason, universes of subjects are also referred to as **universes of infinite subjects or options**, highlighting both their **possible infinite nature** and their treatment as options within probabilistic models.

Even when a universe is not infinite in a longitudinal sense of time, the **subdivision of time** between any two moments is itself infinite. Thus, within any finite time span, it would still be possible to carry out **infinitely many measurements** of a subject or option. The possibility of infinite subdivision ensures that the idea of "infinite universes" retains practical significance in both the natural and social sciences.

37.22. Universes of Subjects Versus Universes of Options

The reason for emphasizing the possible existence of infinite universes is to establish their contrast with **universes of options**.

- **Universes of subjects** are structured around **qualities** that can vary in intensity across subjects.
- **Universes of options**, by contrast, are **directly bounded** by the finite number of alternatives available within the empirical model.

In *Impossible Probability*, these are called **universes of limited options**, precisely because the number of options cannot exceed the bounds imposed by the empirical framework. For example, a subject like a star may have infinite measurable qualities (temperature, mass, luminosity), but when a model specifies discrete options such as "binary system" versus "solitary star," the set of options is strictly limited.

37.23. Universes of Limited Options

Universes of limited options can be further subdivided into **materially limited** and **socially limited** universes.

- **Materially limited universes** arise when physical reality constrains the number of alternatives. For instance, when tossing a coin, the result can only be **heads or tails**; similarly, in human biology, sex assignment has traditionally been modeled in terms of two discrete categories, **male or female**.
- **Socially limited universes** occur when social conventions, institutions, or policies determine the set of available alternatives. For example, in a democratic election, the distribution of voting intentions is confined to the finite set of **candidates or parties** that participate.

[Other limited universes could be discrete categories established by the scientific policy, or another one technological universes]

The different models of universes of limited options are analyzed in greater detail in Section 9 of *Introduction to Impossible Probability*.

Importantly, in these universes, because the set of options is already **fully specified and closed**, the **entire universe of options can be studied directly without the need for prior sampling**. In such cases:

- The **N-sample** coincides with the **total set of options**.
- The **universe of options** itself constitutes the sample, and analysis proceeds by evaluating the **frequency distribution** of those options.

37.24. Object of Study in Universes of Options

In **universes of subjects**, the focus of study is the *direct score* associated with each subject individually—that is, the measurement of a quality's intensity in each case. By contrast, in **universes of options**, what is studied is the **distribution of frequency** across the available alternatives.

The **frequency of an option** refers to the number of times that a particular option occurs within the selected sample. Thus, if the direct score expresses the **magnitude of intensity of a quality** within a subject, frequency expresses the **magnitude of intensity of the number of occurrences** of an option. Both direct scores and frequencies can therefore be seen as distinct yet equivalent ways of expressing **intensity**:

- In the case of the direct score, it is the intensity of a quality.
- In the case of frequency, it is the intensity of occurrence.

37.25. Redefinition of Direct Score and Frequency

Impossible Probability proposes a **synthetic redefinition** of direct score and frequency, treating them as **essentially identical** in nature: both are measures of **magnitude of intensity**, though expressed differently.

- The **direct score** corresponds to the *qualitative intensity* of a property of the subject.
- The **frequency** corresponds to the *quantitative intensity* of occurrences of an option.

From this reconceptualization, it becomes possible to formulate a **unified theoretical approach** to the study of what is here termed *statistics of probability or probabilistic statistics*. This unified approach means that:

1. **Statistics applies to probability**, insofar as statistical measures of distribution are used to construct probabilistic models.
2. **Probability applies to statistics**, insofar as probabilistic reasoning provides the framework for interpreting statistical variation.

Regardless of whether the study concerns **direct scores in universes of subjects** or **frequencies in universes of options**, the principle remains the same:

- **Empirical probability** = individual direct score or frequency ÷ total direct scores or frequencies.
- **Arithmetic mean** = inverse of N (1/N), with further functions attached to the **multifunctionality of the inversion of N**.

37.26. Universality of Empirical Probability

The reason why empirical probability is universally defined as the **ratio of a direct score or frequency to the total of direct scores or frequencies** is to ensure its validity across all possible types of universes.

- In **universes of subjects**, empirical probability is given by the ratio of a subject's direct score to the total of all direct scores in the sample.
- In **universes of options**, empirical probability is given by the ratio of the frequency of a particular option to the total frequency across the sample.

By synthesizing the two concepts—direct score and frequency—under the broader notion of **intensity magnitude**, *Impossible Probability* achieves a **generalizable measure** applicable to both subjects and options. In essence:

- Direct score = intensity of quality.
- Frequency = intensity of occurrence.

Both result from **statistical measurement**, making them two sides of the same coin. Thus, every kind of universe—whether of subjects or options—can be studied simultaneously as **statistics and probability**.

37.27. The Arithmetic Mean as the Inversion of N

The arithmetic mean can also be reinterpreted within this framework. Since it represents the **average of empirical probabilities**, the arithmetic mean of those probabilities is equal to the **inverse of N (1/N)**.

This holds universally:

- For **N subjects** in a sample drawn from a universe of subjects.
- For **N options** in a universe of options.

Therefore, the **inverse of N** serves a **dual role**:

1. It is the arithmetic mean of empirical probabilities.
2. It is the **theoretical probability**, representing the baseline expectation in probabilistic models.

In *Impossible Probability*, the inversion of N thus emerges as a **multifunctional operator**, simultaneously grounding descriptive statistics and inferential probability in a unified theoretical framework.

37.28. Universal Statistical Treatment

In this framework, whether in **universes of subjects**—where direct scores are measured for each individual subject—or in **universes of options**—where the frequency of each option is measured—the **statistical treatment of probability is ultimately the same**.

Both types of universes converge on a unified methodology in which **empirical probabilities** (derived from observed data) and **theoretical probabilities** (derived from mathematical models) are processed within the same logical structure.

Within this unification, the traditional constructs of classical statistics are reformulated:

- The **differential score** of traditional statistics is redefined as the **Bias Level** (*Nivel de Sesgo*), representing the difference between empirical and theoretical probabilities.
- The **typical or Standard score** is likewise redefined to adapt to the probabilistic framework, serving as a measure that links statistical deviation with probability-based reasoning. [In Impossible Probability, Typical or Standard score is Bias Level divided by Standard Deviation]

Thus, *Impossible Probability* transforms statistical analysis into a **universal approach**, valid for both direct scores and frequencies, integrating the descriptive power of statistics with the inferential depth of probability. This synthesis, developed in detail in Section 15 of *Introduction to Impossible Probability*, provides the groundwork for a consistent and logically coherent system applicable across all forms of empirical research.

37.29. Redefinition of Basic Concepts

By adopting a perspective that **radically reexamines the most fundamental concepts of statistics and probability**, *Impossible Probability* proposes not merely a minor adjustment but a **structural redefinition** of the disciplines themselves.

This redefinition results in a new **mathematical discourse**, distinct from traditional treatments, which seeks to address the **emerging challenges and demands of contemporary science**. Modern scientific inquiry, particularly within chaotic, nonlinear, and complex systems, requires conceptual tools capable of transcending the rigid boundaries that once separated statistics and probability.

The reformulation of concepts such as **direct score**, **frequency**, **empirical probability**, and **theoretical probability** creates the foundation for a **unified field of study**: the *statistics of probability* (or *probabilistic statistics*). This field does not merely complement classical approaches but provides new tools, offering a framework where descriptive and inferential dimensions are integrated into a single coherent system.

Through this process, *Impossible Probability* situates itself as both a **critical response to the limitations of traditional statistics** and a **positive proposal for the future of mathematical science**, aligning itself with the broader philosophical traditions of critical rationalism, positivism, and dialectical materialism, while also engaging the methodological challenges of contemporary empirical research.

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38. Subject or Option

[The reason why there are two posts regarding ‘subject’ or ‘option’ is that, on the 8th of December 2012, I transformed the blog *Probabilidad Imposible* into an encyclopaedia of Impossible Probability. On that day, I wrote short posts on different topics, and later I rewrote those posts in more detail. This is therefore the second post on this topic. In other cases, the shorter post was replaced by the longer one.]

38.1. Definition of Subject or Option in Statistical Probability

The term *subject* or *option* designates the individual element of study within *statistical probability*—a conceptual field that in *Impossible Probability* seeks to unify classical statistics and probability theory. Such an element can belong to a universe that is either limited or infinite. From this universe, a sample is formed, or, in the case of a population study, the entire population is considered as the object of analysis. In this latter case, the population itself is treated as if it were a sample of its own historical trajectory. Each individual element—whether referred to as *subject* or *option*—is measured through either its direct scores or its frequencies, and from these measurements one can estimate, a posteriori, its empirical probability in relation to the entire sample.

This definition highlights a fundamental methodological step in *Impossible Probability*: the transition from raw measurements (direct scores or frequencies) to a probabilistic framework where each element is situated within the larger distribution of the sample. Thus, the empirical probability of a subject or option is not assumed a priori, but derived from measurement and statistical evaluation.

38.2. Synthesis Between the Statistical Subject and the Probabilistic Option

Statistical probability, as elaborated in the theory of *Impossible Probability*, constitutes a new scientific field that synthesizes the domains of classical statistics and traditional probability. One of its conceptual breakthroughs is the unification of the *statistical subject* and the *probabilistic option*. In this framework, every subject is simultaneously treated as a possible option, and conversely, every option is regarded as a subject.

This synthesis resolves a long-standing epistemological division: statistics traditionally concerns itself with *subjects* of measurement (observations, individuals, or experimental units), whereas probability theory deals with *options* or possible outcomes of an event. In *Impossible Probability*, these two perspectives are reconciled within a single conceptual structure, enabling a more flexible approach to empirical reality where the same element can be analyzed under both statistical and probabilistic perspectives.

38.3. Definition of Option and the Concept of a Sample of Options

An *option* is defined as a possible alternative among several possibilities, though not necessarily the only one. The empirical frequency of an option is calculated as the number of occurrences in favor of that particular option divided by the total number of options. This total number of options, denoted by *N*, is called the *sample of options*.

A sample of options defines a *universe of limited options*. Within such a universe, the options are strictly predetermined by the empirical model itself: there can be no more or fewer options than those that constitute the sample N. Consequently, the empirical probability of each option is equal to its specific frequency divided by the total frequency of all options combined. This guarantees that the total probability across all options sums to unity.

From this perspective, options are inherent to the empirical structure of the phenomenon [or by social structures, or scientific definitions, or technological options]. The role of science is to measure their relative frequencies and interpret their probabilities within the logic of the model.

38.4. Types of Universes of Limited Options

As developed in Section 9 of *Introduction to Impossible Probability*, universes of limited options can be classified into two broad categories, depending on whether the definition of *N* (the total number of options) is determined by the empirical hypothesis or allowed to vary as a parameter under scientific policy.

- **Universes of strictly limited options** are those where the value of *N* is fixed by the empirical hypothesis itself. These can be further subdivided into:
    1. **Materially limited universes** (e.g., a coin toss can only result in "heads" or "tails"; the biological sex of humans is traditionally defined as male or female).
    2. **Socially limited universes** (e.g., the distribution of votes among political parties in an election, where the number of parties is predefined).
- **Universes of variably limited options**, by contrast, are those in which the value of *N* is not rigidly tied to an empirical hypothesis. Instead, *N* can be modified arbitrarily by scientific policy, such as in models involving *discrete categories* where new categories may be introduced, merged, or eliminated as research evolves.
- **[A new category is universes limited to technological options]**

This distinction allows *Impossible Probability* to extend beyond rigid empirical models, incorporating both natural and social systems, and acknowledging the role of human policy and scientific convention in shaping the boundaries of the universe under study.

38.5. The Abstract Concept of Subject

The concept of *subject* should not be understood in a limited or restricted sense as a particular or concrete person, as it is commonly used in colloquial language. Instead, *subject* must be understood in its most abstract or morphological sense: the individual reference element of a predicate, that which is being predicated. In statistics, or in mathematics more generally, the direct score is the predicate of the subject. In other words, the *subject* is any individual referential element of a direct score.

Since the theory of Impossible Probability synthesizes statistics and probability, the *subject* will be treated as an *option*, in the same way that an *option* will be treated as a *subject*. If the sample of options is the sample of *N* options, the sample of subjects will be the sample of *N* subjects. In the same way that the empirical probability of an option is calculated, the empirical probability of a subject can be estimated. However, instead of being based on frequencies, it will be based directly on direct scores. Thus, the empirical probability of a subject will be equal to the individual direct score divided by the sum of all direct scores.

38.6. Empirical Probability of Subjects and Options

Since the empirical probability of an option is equal to the particular frequency divided by the total, and the empirical probability of a subject is equal to the individual direct score divided by the total, then in general terms—and in order for the definition of empirical probability to be valid and universal for any type of universe, whether of subjects-as-options or of limited options—in Impossible Probability it will be stated that the empirical probability of a subject or option is equal to the direct score or frequency divided by the sum of direct scores or frequencies.

Studies based on direct scores normally refer to universes of subjects-as-options, which will be called *universes of subjects or infinite subjects or options*, or simply *infinite universes*. By contrast, studies based on frequencies normally refer to universes of limited options. Exceptions may exist, for example, in certain universes of limited options corresponding to discrete categories, or other possible phenomena that cannot be ruled out, in which case universes of limited options may transform into universes of infinite subjects.

38.7. Difference Between Universes of Limited Options and Infinite Universes

In Impossible Probability, one of the main differences between universes of limited options and universes of subjects or infinite subjects or options is that universes of options are normally finite, while universes of subjects or options may be infinite. This is why, in *Introduction to Impossible Probability*, they are called *universes of infinite subjects or options,* or simply *infinite universes*.

The reason for affirming that these universes may be infinite lies in the principle that matter and energy are neither created nor destroyed. The logical deduction is that both matter and energy are either timeless, eternal, or infinite. In other words, what cannot be destroyed necessarily has no chronological end, and by definition, what has no end is infinite.

[As Kant observed, the debate about the existence of the infinite is itself an antinomy, for there are as many arguments in favor as against this thesis. In this debate, our position is clear: the infinite exists. The problem does not lie in the notion of the infinite but in human limitation. For a finite mind, the very idea of the infinite inevitably appears contradictory]

38.8. Cosmological Possibility of Infinite Universes

From a cosmological perspective, the cycles of transformation between matter and energy may be infinite. This possibility entails scenarios such as endless cycles of entropy, infinite sequences of cosmic expansion and contraction driven by gravity, or the existence of stationary states of matter and energy that persist without limit. Beyond these cyclic processes, one can also conceive of *infinitely many histories of our universe* unfolding across time, or the existence of *multiple universes*, either successive in sequence or simultaneous in parallel.

In this framework, the infinity of universes is not merely a speculative hypothesis, but a logical consequence of considering matter and energy as fundamentally indestructible and thus capable of undergoing limitless transformations. Such a conception places probability theory in dialogue with cosmology, since infinite cycles and universes imply that what may seem improbable—or even impossible—within a finite frame becomes inevitable when extended to infinity.

38.9. Kant's Antinomy of the Infinite

The difficulty in grasping the concept of infinity gives rise to what Kant described as *logical antinomies*. These occur whenever human reason advances suppositions through intuition or deduction without the support of empirical evidence. The infinite thus becomes a field where equally convincing arguments can be formulated both in favor and against its existence.

Because empirical verification is absent, these antinomies cannot be resolved positively: we lack material evidence to either confirm or disprove the infinite. Kant's insight underscores that infinity lies at the boundary of human cognition, where rational thought encounters the limits of empirical justification. This epistemological challenge persists in modern science, where infinity continues to serve both as a powerful heuristic and as a reminder of our cognitive limitations.

38.10. Impossible Probability and Critical Rationalism

The theory of *Impossible Probability* situates itself within the tradition of *critical rationalism* and accepts Kant's hypothesis that *reality-in-itself* (*das Ding an sich*) is ultimately unknowable. Human knowledge is limited to *phenomena*—that is, the manifestations of reality available to our senses and instruments.

Within the framework of *statistical probability*, this translates into a methodological principle: our knowledge of reality is always mediated by samples, from which we construct logical deductions. The process involves the rational critique of positive data,

the testing of hypotheses, and their acceptance or rejection within defined margins of error.

Furthermore, drawing on dialectical materialism, *Impossible Probability* recognizes that this rational critique is not purely abstract but essentially *political*. Scientific policy (or "política científica") is understood as the collective framework within which critical reason operates, establishing the acceptable margins of error and thus shaping the provisional truths that science can endorse.

38.11. The Question of Infinity in Impossible Probability

The issue of infinity is explicitly addressed in Section 7 of *Introduction to Impossible Probability* and plays a central role in defining *universes of infinite subjects or options* (also referred to simply as "infinite universes"). The distinction arises because universes of limited options, by definition, cannot tend toward infinity—except in certain special cases outlined in Section 9.

However, if one accepts the possible infinitude of matter and energy across space–time, then several scenarios become conceivable:

- an infinite universe,
- infinitely many histories of a single universe,
- infinitely many universes, either successive or parallel,
- or even infinitely many histories of infinitely many infinite universes.

Under these assumptions, universes of subjects are no longer confined to finitude. They may indeed be *infinite universes of subjects*, where each subject represents one possible element within an *N* that tends toward infinity. This perspective aligns the mathematical study of probability with cosmological speculation, while also reinforcing the critical rationalist stance: infinity cannot be proven or refuted, yet its possibility must be integrated into the logic of probability, since the acceptance of infinity profoundly alters the way we conceive error, chance, and inevitability.

38.12. Infinity as the Genesis of Error

Within the tradition of critical rationalism, the possibility of infinity cannot be ruled out. If this hypothesis is accepted—as postulated in *Impossible Probability*—then infinity becomes the very *genesis of error*. As explained in Section 7 of the *Introduction*, the fundamental source of error lies in the contradiction between an infinite reality and the finite ***limitations of human cognition*****.**

Because human beings cannot know everything, they must necessarily accept margins of ignorance, margins of error, margins of skepticism, or margins of doubt. This produces a permanent state of relative uncertainty in which everything remains possible. Within such a framework, empirical hypotheses that are accepted as provisionally rational may later prove false. Conversely, within the same margins of ignorance, what was once thought to be impossible may in fact reveal itself as inevitable.

This duality—the potential falsity of all knowledge and the inevitability of the impossible—leads toward a form of *relative nihilism*. In this view, the very possibility of absolute knowledge dissolves into the margins of doubt. Yet, paradoxically, science continues to advance at an accelerated pace, experiencing an exponential increase in successive crises and scientific revolutions. Provided that these crises are resolved positively, they point toward a dynamic equilibrium—a *state of permanent revolution in science*.

[While working on the blog, I developed an extreme nihilist vision: if, in the end, all science is provisional, then nothing is true—this seems the logical conclusion. Yet this nihilism is dispelled as soon as we begin working on Global AI. Our hope is clear. At first, a Superintelligence might be ten times more intelligent than Einstein and Heisenberg combined. But at some point, this Superintelligence could become a hundred times more intelligent, and perhaps even a thousand times more intelligent. If such acceleration continues, our hope is that the Superintelligence will evolve into a Global

AI. At that stage, for such an intelligent machine, the development of non-human mathematics—and therefore non-human pure operations—could become not only possible but inevitable. A Global AI, free from human limitations and endowed with immense intelligence capable of performing pure operations far beyond our reach, could open the pure knowledge of the universe, like a Pandora's box.]

38.13. Limited Universes with Infinite Samples of Frequencies

The consideration of infinity also highlights the narrow boundaries of scientific inquiry. Strictly speaking, the term *infinite universe* refers to universes of infinite subjects or options. However, even universes of *limited options*—whether materially limited, socially limited, or defined by an arbitrarily variable *N [or limited technological universes]*—may nonetheless contain *infinite samples of frequencies*.

For instance, a coin constitutes a universe of limited options: only two alternatives exist, heads or tails. Yet, if one were to toss the coin an infinite number of times to study its outcomes, the *sample of frequencies* would tend toward infinity. Thus, while the coin itself belongs to a limited universe of options, the frequency distribution of outcomes effectively generates an *infinite universe of frequencies*.

This example illustrates a broader principle: limited universes may still be connected to infinite processes of measurement. Just as universes of direct scores may tend toward infinity when measured indefinitely, universes of limited options can give rise to infinite distributions of frequencies. The boundary between the finite and the infinite is therefore less rigid than it first appears, demanding a flexible theoretical framework to reconcile both.

38.14. Identity of Subject/Option and Direct Score/Frequency

The statistical treatment of subjects and options is essentially the same, just as the statistical treatment of direct scores and frequencies is identical. In *Impossible Probability*, this equivalence is framed through the lens of Hegelian dialectics, which posits that *opposites are identical*.

Thus, the *identity of subject and option* stems from the Hegelian notion of the identity of subject and object, while the *identity of direct score and frequency* reflects the Hegelian identity of quality and quantity. The implication is that what appear to be distinct categories in traditional statistical frameworks are, in fact, two sides of the same coin—logical opposites that converge when examined through a dialectical lens.

This synthesis challenges the classical separation between subjects and options, as well as between direct measurements and frequency distributions. By recognizing their dialectical identity, **Impossible Probability provides a more unified statistical language, one that integrates individual qualities (captured through direct scores) with collective occurrences (expressed through frequencies).**

38.15. Critical Application of Hegelian Dialectics

The application of Hegelian dialectics to the field of statistical probability—or *statistical probability theory*—is not carried out in a naïve or dogmatic fashion, but rather in a critical one. As is evident throughout the *Introduction to Impossible Probability*, the identity of opposites is admitted only within the boundaries of the margin of error that scientific policy is prepared to accept.

This has a decisive implication: the isomorphism between idea and reality—between empirical hypothesis and empirical world—can only exist in a restricted form. An idea of reality, expressed in the form of a hypothesis, becomes truly rational and thus "identical" to reality only insofar as it holds within the critical margin of error admitted by the scientific community. Beyond this limit, the supposed identity collapses. In other words, dialectical identity does not mean absolute equivalence but conditional equivalence, always mediated by the epistemological framework of error acceptance.

38.16. Margin of Error and the Validity of Empirical Hypotheses

From this standpoint, the identity of *subject and option* in universes of infinite subjects or options is only possible within the limits defined by the margin of error accepted by scientific policy. Mathematically, this margin of error is expressed as the *inverse of N*

(1/N), which represents the probability of sampling error in universes of infinite subjects or options.

Similarly, the identity of *direct score and frequency* in universes of limited options is valid only within the corresponding margin of error. In these cases, the measure of error is given by the *inverse of the sum of direct scores or frequencies* (1/Σxi). Thus, both in infinite universes and in limited ones, the possibility of treating opposites as identical depends entirely on the structural limits imposed by the accepted error framework.

38.17. Probability of Error in Limited Universes

The reason why the inverse of direct scores or frequencies (1/Σxi) constitutes the probability of sampling error in universes of limited options lies in the nature of such universes: the set of possible options is determined by the universe itself. Therefore, the true locus of randomness and error is not in the definition of options—which are fixed (e.g., heads or tails, wave or particle, male or female)—but in the distribution of frequencies within the sample.

For example, in studying the probability of a coin landing on heads or tails, the options are predetermined: there are only two. Likewise, in quantum physics, a particle may behave as a wave or as a particle under given conditions, and in sociology, the proportion of men and women in a society is structurally limited to those two categories. In all such cases, the randomness that generates statistical error does not derive from the set of options but from the variability in their observed frequencies across samples.

38.18. Validation of Hypotheses within the Margin of Error

It is precisely within the margin of error—whether factual (1/N in infinite universes or 1/Σxi in limited universes) or critical (the critical reason margin set by scientific policy)—that hypotheses are validated or refuted as rational. Within this margin, *everything is possible*: an empirical hypothesis accepted as rational may nonetheless be false, and phenomena previously dismissed as impossible may in fact prove inevitable.

This conception reshapes the philosophy of science into one of permanent critical vigilance. The rationality of hypotheses is always provisional, resting on the fragile equilibrium between empirical evidence, logical critique, and the institutional margins of error determined by scientific policy.

38.19. Subject or Option as Reference Elements

Ultimately, *subject or option* serve as the individual reference elements in statistical probability theory. It is upon them that empirical probabilities are calculated, derived from direct scores (in the case of subjects) or frequencies (in the case of options). By comparing *empirical probability* with *theoretical probability*—which in all universes, finite or infinite, is represented by 1/N as the arithmetic mean—we can estimate the *Bias Level* (*Nivel de Sesgo*) and construct the other measures of dispersion, such as Mean Deviation, Variance, and Standard Deviation.

These metrics form the foundation for the rational critique of hypotheses. Whether applied intrasample (as explained in Section 11 of the *Introduction to Impossible Probability*) or inter-samples (as elaborated in Sections 16–19), they provide the statistical machinery through which provisional knowledge is tested, corrected, and either consolidated or replaced.

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On 10 August 2014,
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39. Samples

39.1. The General Concept of a Sample

A sample is, in its most elementary sense, a form of proof. It can be a proof of emotions, a demonstration of virtues, or a confirmation of truth. It is a proof of emotions when an individual communicates their feelings openly; it is a proof of virtues when someone showcases a skill or ability; and it is a proof of truth when, in order to demonstrate the validity of a statement, we present concrete evidence that substantiates it. In every case, the notion of a sample points toward the same underlying principle: it is an act of showing or displaying something before others as evidence—evidence of what we feel, what we are capable of doing, or what we think.

From a philosophical and epistemological perspective, the essence of a sample lies in its representational function. A sample is never merely a fragment of a whole but an intentional selection designed to serve as evidence. Its meaning is relational: it only ***acquires value when placed before another subject—an observer, a community, or a scientific tribunal—who recognizes its representative or demonstrative character***. Thus, the concept of the sample transcends the empirical act of selection and becomes an act of communication between the self and others, between empirical reality and rational judgement.

39.2. The Sample in the Scientific Domain

In the sciences, the concept of a sample takes on a more rigorous and technical meaning. A sample is understood as a test or piece of evidence that verifies the truth of a proposition. This proposition may arise inductively, as a generalization from a collection of empirical facts, or deductively, as a hypothesis subject to rational critique. In either case, the collection of empirical facts or the series of phenomena under

examination is termed a *sample*, insofar as it is that sample which demonstrates the logical conclusions of induction or enables the verification of an empirical hypothesis formulated by a scientist or a team of scientists.

Thus, the sample in science is not merely a fragment of reality but a methodological instrument. It is a bridge between empirical reality and theoretical reasoning. The act of sampling introduces a critical dimension: the reliability of the conclusions reached depends on the quality, representativeness, and internal coherence of the sample. A sample, therefore, functions as a mediator in the epistemic process of science: it mediates between the infinite complexity of reality and the finite, structured form of knowledge.

39.3. The Sample in Quantitative Epistemology

Within the framework of quantitative epistemology, one of the principal methods for analyzing data sets is the use of statistics and probability. The fusion of these two disciplines in the theory of *Impossible Probability* gives rise to a new field: the *statistics of probability* or *statistical probability*. In this theoretical system, the sample assumes a central role, since it is through the construction and analysis of samples extracted from the universe that rational critique of reality can be pursued.

The sample becomes the operational unit of rational criticism. It is the object upon which *empirical measurements are made and the vehicle through which reality is transformed into a set of analyzable values*. From this perspective, sampling is not only a technical necessity but also a philosophical act: it embodies the reduction of empirical infinity into a finite and manageable subset that can *be subjected to rational operations*. *In Impossible Probability*, this act of sampling acquires heightened epistemological significance, since the reliability of critical rationalism depends on the rigor of the constructed samples and on the logical consistency with which they are treated.

39.4. The Function of the Sample in Descriptive and Inferential Statistics

Consequently, *the sample can be defined as the set of events or occurrences whose distribution allows different forms of statistical reasoning*. In descriptive statistics, the distribution of the sample leads to the construction of a description of a given reality. In inferential statistics, by contrast, the sample is used to perform hypothesis testing, where the critical assessment of tendencies within the data is carried out.

The dual role of the sample illustrates its versatility: on the one hand, it provides an empirical image of reality (descriptive function), and on the other, it provides the basis for critical rational evaluation of hypotheses (inferential function). In both contexts, the sample is more than an empirical subset: it is the foundation for the methodological interplay between observation and theory, between empirical description and logical critique.

39.5. Types of Universes and Samples in the Second Method

Within the Second Method of Impossible Probability, a more nuanced typology of universes and samples is introduced. Two types of universes are distinguished: infinite universes and limited universes. Correspondingly, two types of samples are defined: the *sample of subjects or options* and the *sample of direct scores or frequencies*. Each of these samples is associated with particular forms of statistical reasoning and specific kinds of theoretical-statistical measures, both individual and collective.

The innovation of Impossible Probability lies in recognizing that the interpretation of a sample depends fundamentally on the type of universe to which it belongs. In limited universes, the emphasis falls on the distribution of frequencies across a finite set of options; in infinite universes, the focus shifts to the tendencies of direct scores across potentially unbounded populations. The inversion of these samples—expressed mathematically as reciprocals (e.g., 1/N or 1/Σxi)—constitutes the basis for deriving theoretical statistics, both individual and sample-based, within the different universes.

In this sense, the Second Method not only reformulates the meaning of a sample but also redefines the epistemological framework in which statistical reasoning operates. The sample ceases to be a passive reflection of empirical reality and becomes an active theoretical construct, indispensable for bridging the gap between probability and statistics, between empirical induction and rational critique.

39.6. Universes of Limited Options

Universes of limited options are defined as those in which the distribution of frequencies is studied over a finite set of possible options. The number of options establishes the magnitude **N**, such that the sample is composed of these **N** options. These options may be limited by natural constraints, by social conventions, or by scientific-policy decisions in cases where **N** is considered a variable magnitude, [In technological limited universes N is limited by the technological options].

The notion of a limited-option universe highlights the fact that the scope of the sample is not infinite but predetermined. This predetermined nature has epistemological consequences: it forces the researcher to work within a closed system of possibilities, where the representativeness of the sample is tied to the completeness of the option set. For example, when the universe is naturally limited, every option is known in advance; when socially limited, the recognition of options depends on institutional or legal frameworks; and when politically limited, the scientific community itself defines the scope of options for methodological reasons. In all cases, **N** provides a structural anchor for the statistical model.

39.7. Examples of Universes of Limited Options

Examples illustrate how universes of limited options manifest in both natural and social contexts. In a natural universe, consider the study of the distribution of chemical elements within a sample: here the universe is strictly limited by the options of the periodic table. Each element is an option, and the number of options corresponds to the number of recognized elements. In a social universe, an analogous case arises

when studying the distribution of legal forms of private companies. The available options—sole proprietorship, limited liability company, joint-stock company, or any other legal form defined by law—constitute a socially bounded universe.

There are also cases of **variable magnitude N**, particularly in studies involving discrete categories. Here the number of categories depends on the criteria established by scientific policy, yet the categories must always remain sufficiently representative and significant to ensure the reliability of the study. As explained in section 10 of *Introduction to Impossible Probability* (updated in January 2015), this requirement prevents arbitrary or trivial classifications. For instance, in a study of income distribution across a population, categories could be constructed based on annual income levels. Each subdivision is one among an infinite number of possible categorizations, but the chosen division determines the way social class membership is defined within that specific universe.

These examples emphasize that universes of limited options are not only empirical in character but also epistemologically dependent on the boundaries chosen for their definition.

39.8. Fundamental Difference Between the First and Second Methods

One of the principal differences between the Second Method of Impossible Probability and what would be considered traditional statistics (called the *First Method* within Impossible Probability, as explained in section 4 of *Introduction to Impossible Probability*) lies in the definition of **N**. In Impossible Probability, **N** refers to the *number of subjects or options*, whereas in traditional statistics, **N** refers to the *total frequency*.

This distinction is essential, because the way **N** is defined influences every subsequent definition of both statistics and probability. If **N** is understood as the number of subjects or options, statistical reasoning focuses on the distribution of probabilities across possible outcomes. If **N** is understood as total frequency, then statistical reasoning shifts toward observed occurrences within a dataset. The conceptual framework of Impossible Probability thus departs fundamentally from traditional statistics: it begins

with the logical space of subjects or options rather than the empirical tally of frequencies.

The redefinition of **N** therefore reconfigures the entire epistemological foundation of probability and statistics, giving rise to new interpretations, operations, and theoretical constructs within the Second Method.

39.9. The True Sample Selection in Limited Universes

In universes of limited options, the Second Method distinguishes between two levels of sampling. The sample of options **N** is a predefined set, determined either by the empirical model (natural or social) or by scientific policy [or technologies]. However, this sample of options is not the true selection sample. The true selection is represented by the **sample of direct scores or frequencies**, denoted as **Σxi**.

For example, if we study whether a coin lands heads or tails, the true selection is not the set of two possible outcomes (heads and tails) but the actual number of coin tosses performed in the study. Similarly, in a study of the distribution of workers by professional category, the true selection is not the list of possible job categories but the actual number of workers included in the analysis. This may involve a single company, a national population, or even the global workforce. Each direct score or frequency is designated by the symbol **xi**, and the total sample is the summation of these frequencies, **Σxi**.

This distinction clarifies that, in limited universes, representativeness does not arise solely from the definition of options but from the accumulation of empirical data points within those options. The theoretical implications are significant: while the options provide the logical framework, it is the direct scores and their summation that make statistical analysis operational and meaningful.

39.10. Universes of Infinite Subjects or Options

Universes of infinite subjects or options are those in which the distribution of direct scores is studied across an unbounded set of cases. These direct scores correspond to the measurement of a particular quality or attribute within the universe under investigation. In such models, individual subjects are treated statistically as *options*, which is why they are referred to as *universes of subjects or options*. By definition, the hypothesis is that these universes tend toward infinity, either because the potential set of cases is not predetermined or because new cases can always be added in principle.

In these universes, the sample **N** is the set of **N subjects or options**, and the act of sample selection becomes indispensable. Even in cases where an entire population appears to be under study—such as in demographic censuses—the population itself functions as a sample. This is because what is selected is not the entirety of humanity across all time, but a population defined within a specific historical and temporal context. Hence, a national census in the year 2025 is, in fact, a sample of that population's behavior at that particular moment in history.

The distinction between limited-option universes and infinite-subjects/options universes is crucial but not absolute. Whether in limited natural universes, socially defined universes, or infinite universes of subjects, the principle remains: **N always represents the sample of subjects or options**. What changes is the scale of the universe and the epistemological assumptions regarding its boundaries. Limited universes are inherently closed; infinite universes are open-ended and hypothesized to extend without final boundary.

39.11. The Definition of N and Theoretical Statistics

The definition of **N** is the key that unlocks the construction of theoretical statistics. Because **N** acquires a multifunctional value, it applies simultaneously to universes of limited options and to universes of infinite subjects or options. Regardless of the type of universe, the inversion of **N** (expressed as 1/N) is equated with the arithmetic mean of empirical probabilities.

In formal terms, empirical probabilities in any universe are defined as the absolute value of a direct score or frequency (**xi**) divided by the sum of all absolute values of direct scores or frequencies (**Σxi**). By construction, the arithmetic mean of these empirical probabilities is always equal to the inversion of **N**. Thus, **N** is consistently defined as the sample of subjects or options across all universes, whether finite or infinite.

This definition gives theoretical statistics a unified mathematical foundation: by linking **N** to the mean probability across the sample, Impossible Probability ensures that statistical reasoning is anchored in a single invariant principle. The inversion of **N** becomes not only a mathematical ratio but also an epistemological constant: a benchmark that integrates empirical measurement with theoretical expectation.

39.12. The Exception of Zero-Sample Cases

The only exception to the rule that inversion of **N** (1/N) equals the arithmetic mean occurs in what are called **zero-sample cases**. These are situations where every empirical probability in the sample equals zero. This condition is designated in Impossible Probability as a case of *Impossible Probability* itself, since it represents the complete absence of positive empirical occurrence.

Such cases can arise even in universes of infinite subjects or options, particularly when the quality measured by the direct scores is defined as a negative or non-ideal attribute. Consider, for example, a medical study of a new treatment. ***Suppose a drug is discovered that cures a specific disease.*** To demonstrate its efficacy empirically, researchers would need to show that among a sample of **N** patients, the empirical probability of symptoms per patient is reduced to zero after treatment. In this situation, every patient has a zero probability of exhibiting symptoms. This constitutes a **sample of zeros**.

The epistemological importance of zero-sample cases lies in their role as limiting conditions. They reveal the boundaries of traditional statistical reasoning, in which a mean of zero automatically implies zero variance and zero dispersion. By contrast, in

the Second Method of Impossible Probability, zero-sample cases require special handling, since **N** is not itself zero, even though every observed probability is zero. This paradox gives rise to distinctive statistical functions that differentiate Impossible Probability from classical approaches.

39.13. Examples of the Application of Zero-Sample Cases

In universes of limited options, it is possible to encounter situations in which all the available options are considered negative outcomes (i.e. symptoms in subjects experimenting a new medical treatment, errors in a chain production). In such cases, the *ideal* outcome would be zero frequency across all options, since none of the options is desirable. However, when there exists a gradation in the severity of the negative options—meaning that some outcomes are worse than others—the Second Method of Impossible Probability is not always the most appropriate tool for analysis.

Instead, it becomes more effective to apply the **Defect Impact Method**, described in section 21 of *Introduction to Impossible Probability*. This method is specifically designed to analyze the magnitude and consequences of negative outcomes, weighting them according to their degree of severity. Zero-sample cases, in this sense, are not merely mathematical curiosities but methodological signals: they indicate situations in which the analytical framework must shift toward specialized techniques capable of addressing the asymmetry of negative conditions.

39.14. The Real Function of the Inversion of N in Zero-Sample Cases

The reason why the inversion of **N** (1/N) does not function as the arithmetic mean in zero-sample cases lies in the distinction between classical statistics and Impossible Probability. In the traditional (First Method) framework, the arithmetic mean of zero scores or zero frequencies is simply zero, which implies zero dispersion. Consequently, variance and standard deviation also collapse to zero.

In the Second Method of Impossible Probability, however, **N** is not equal to zero even when every empirical probability is zero. Therefore, its inversion—1/N—remains distinct

from zero. As a result, even in zero-sample cases, the **Bias Levels** of each subject or option are defined as equal to the absolute value of 1/N. This implies that both the Mean Deviation and the Standard Deviation are equal to 1/N.

This reconfiguration demonstrates the deeper epistemological innovation of Impossible Probability: statistical measures are not reduced to trivial zero values simply because the observed data are all zero. Instead, they preserve a theoretical structure that continues to quantify dispersion and bias, albeit in a non-classical way.

39.15. The Inversion of N as Theoretical Probability

The reason why the inversion of **N** (1/N) remains non-zero—even in zero-sample cases where the arithmetic mean itself is zero—is that its true function is not reducible to the calculation of averages. While it normally coincides with the arithmetic mean in non-zero samples, its deeper role is to represent the **theoretical probability of occurrence under conditions of equal opportunity by chance**.

This means that if, at any given moment, the distribution of direct scores or frequencies among the subjects or options were entirely random, the empirical probabilities would naturally tend toward 1/N. In this sense, inversion of **N** functions as a baseline probability, an a priori reference point for equal distribution in conditions of chance. Zero-sample cases highlight this function by breaking the coincidence between empirical averages and theoretical probabilities: even when empirical data vanish, the theoretical probability persists as a structural constant.

39.16. Consequences of the Definition of N

The definition of **N** has far-reaching consequences for the entire structure of statistical probability. Depending on whether **N** is defined as the total frequency (as in classical statistics) or as the number of subjects or options (as in Impossible Probability), the subsequent definitions of probability and statistical functions diverge profoundly.

In the traditional framework, where **N** equals the total frequency, the focus remains on observed counts and their distribution. By contrast, in Impossible Probability, **N** is defined as the sample of subjects or options across any type of universe, finite or infinite. From this definition, the first and most universal functions of the inversion of **N** (1/N) follow: it serves simultaneously as the arithmetic mean (except in zero-sample cases) and as the theoretical probability of occurrence under conditions of equal opportunity by chance.

This dual role of 1/N ensures that Impossible Probability bridges descriptive and theoretical statistics in a unified way. It grounds empirical measurement in a constant logical structure while preserving flexibility to handle exceptional cases, such as zero-sample distributions.

39.17. Additional Functions in Infinite Universes

In the specific case of universes of infinite subjects or options, the inversion of **N** (1/N) acquires additional functions beyond those it performs in limited universes. One of these is its role as a measure of **theoretical dispersion probability**. As the size of the sample **N** increases toward infinity, the inversion 1/N approaches zero. Consequently, both the individual dispersion (Bias Level) and the sample dispersion (Mean Deviation) also tend toward zero as **N** tends to infinity.

This reveals a fundamental insight: in infinite universes, randomness itself stabilizes as the sample grows. The law of large numbers is given a new epistemological expression in Impossible Probability: the larger the sample, the smaller the expected deviations, until in the limit of infinity dispersion vanishes. Thus, 1/N becomes a quantitative index for the convergence of empirical distributions toward theoretical uniformity.

39.18. Sample Representativity in Infinite Universes

Simultaneously, in infinite universes, 1/N serves as a measure of the **probability of error in sample representativity**. As the sample size **N** increases and more concrete cases of events or occurrences are included, the sample becomes progressively more

representative of the universe. Accordingly, the probability of representational error decreases in exact proportion to 1/N.

In this context, inversion of **N** plays a dual role: it quantifies not only the expected dispersion of empirical distributions but also the risk that a given sample may fail to represent the population faithfully. As **N** tends to infinity, both dispersion and representativity error converge to zero. In this way, Impossible Probability extends the classical idea of statistical convergence, framing it within a unified system of bias levels and theoretical dispersion.

39.19. Functions in Universes of Limited Options

While in infinite universes theoretical dispersion probability and error of sample representativity are expressed by the inversion of **N** (1/N), in universes of limited options the situation is different. In these cases, the true selection sample is not **N**, but rather the sample of direct scores or frequencies, denoted **Σxi**.

Here, the functions of dispersion and representativity are governed by the inversion of these direct frequencies, **1/Σxi**. Nevertheless, the inversion of **N** (1/N) continues to perform its universal functions: it remains the arithmetic mean of empirical probabilities and the theoretical baseline probability of equal chance distribution, applicable to both limited and infinite universes.

This dual structure highlights one of the central methodological innovations of Impossible Probability: it decouples the logical role of **N** (number of options) from the empirical role of **Σxi** (frequency sample). Each inversion operates within its own domain, ensuring that the system maintains both universal theoretical consistency and adaptability to particular empirical configurations.

39.20. Examples of Dispersion in Limited Universes

The reason why the inversion of direct scores or frequencies (1/Σxi) functions as theoretical dispersion probability in limited universes lies in the dynamics of sample growth. The larger the frequency sample, the smaller the dispersion between options.

For example:

- In repeated coin tosses, the more tosses are made, the closer the empirical probability of heads or tails approaches equilibrium.
- In a maternity ward, as the number of births increases, the probability of observing roughly equal numbers of boys and girls rises.
- In democratic elections, higher voter turnout tends to equalize the relative strength of political options under normal conditions.

These examples illustrate how, in limited universes, frequency-based sampling reduces the variability of empirical outcomes, converging toward balanced distributions. The mathematical expression of this convergence is precisely **1/Σxi**, the inversion of the total observed frequencies.

39.21. Relationship Between Dispersion and Representativity in Limited Universes

In universes of limited options, the reduction of dispersion is directly linked to the representativity of the sample. As the frequency sample grows, empirical probabilities across options tend to converge, making the sample more representative of the universe. Thus, the theoretical probability of dispersion in limited universes is inversely proportional to the size of the frequency sample.

Formally, the inversion of direct scores or frequencies (1/Σxi) equals the theoretical probability of dispersion under normal conditions. This establishes a clear proportionality: the larger the sample, the smaller the expected deviation, and the more accurate the representativity of the sample.

39.22. The Double Function of Frequency Inversion in Limited Universes

At the same time, the inversion of direct scores or frequencies (1/Σxi) performs a double function in limited universes. It not only measures the theoretical probability of dispersion but also quantifies the **probability of error in representativity**.

This means that in studies of limited options, sample representativity is inversely proportional to the size of the frequency sample. The larger the sample of direct scores, the smaller the probability of representational error. Consequently, 1/Σxi becomes the key indicator for both dispersion and representativity within the statistical framework of Impossible Probability.

39.23. Redefinition of Statistical Concepts in Impossible Probability

Taken together, these considerations show that depending on how universes, subjects or options, and samples are defined, the resulting definitions in statistical probability may differ qualitatively and quantitatively from those of traditional statistics. The differences are not merely semantic but structural, producing significant changes in the mathematical operations themselves.

In Impossible Probability, these shifts lead to nothing less than a **redefinition of the traditional concepts of statistics**. By integrating universes of limited and infinite options, samples of options and frequencies, and the dual functions of inversion (1/N and 1/Σxi), Impossible Probability achieves a methodological synthesis of probability and statistics. This synthesis transforms statistical reasoning from a merely descriptive or inferential tool into a comprehensive epistemological framework that unifies empirical observation with rational critique.

Rubén García Pedraza, Madrid 14 de febrero del 2015

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40. Theoretical Probability

40.1. Concept of Theoretical Probability and Stochastic Chance

Theoretical probability can be defined as the **hypothetical statistical probability under conditions of equal opportunity in the absence of bias**. A phenomenon or event is said to be random—that is, the product of chance, or *stochastic*—when its occurrence is not determined by any underlying causal mechanism other than chance itself. Such an event is therefore indeterminate: it cannot be predicted with absolute certainty.

When an occurrence takes place randomly, it is described as stochastic in character because its precise manifestation cannot be anticipated in advance. What can be established, however, are **probabilistic margins of error** within which empirical hypotheses or predictions may or may not hold. This distinction between certainty and probability highlights a central theme of *Impossible Probability*: randomness is never absolute chaos, but rather an indeterminate order structured through error margins and probabilistic reasoning.

[The sentence 'randomness is never absolute chaos' is especially important. From the perspective of Paranormal Statistics I developed between 2002-2004, chance may produce levels of bias randomly. A clear example is when the sample size *N* in infinite universes, or the sample of frequencies in limited universes, is not sufficient to be considered representative. Under such conditions of representativeness error, the observed bias is clearly the product of random behavior. In other words, Paranormal Statistics becomes possible under certain conditions—for instance, when there is a lack of a representative sample.]

40.2. Difference Between Theoretical and Empirical Probability

In *Introduction to Impossible Probability*, theoretical probability is developed in explicit contrast to **empirical probability**. This distinction mirrors the differentiation between theoretical and empirical reality.

- **Theoretical probability** is a *hypothetical* form of statistical probability, derived under the assumption of perfect equality of opportunities and the absence of bias.
- **Empirical probability**, on the other hand, is a *real* statistical probability, constructed from actual observations and measurements of phenomena.

Both are fundamental concepts within the framework of *statistical probability* (or *probability statistics*) as elaborated by the theory of Impossible Probability. The dialectical tension between these two notions—one hypothetical, the other real—serves as the basis for rational critique: by comparing empirical distributions with their theoretical counterparts, it becomes possible to identify the presence and magnitude of bias.

40.3. Formal Definition of Theoretical Probability

Formally, the **theoretical statistical probability** of a subject or option, under conditions of equal opportunity and in the absence of bias, is equal to the quotient of unity divided by the number of subjects or options in the sample.

In the framework of Impossible Probability, the total magnitude of subjects or options is denoted by **N**, which represents all possible subjects or options in the sample. The quotient **1/N** is referred to as the *inversion of N*, and one of its key functions is to calculate theoretical probability.

$$\text{Theoretical Probability} = \frac{1}{N}$$

The definition is both elegant and universal: it applies to any kind of sample, whether limited or infinite, natural or social [or technological, a subject could be a cyborg program, or different options could be different types of brain interface], as long as the principle of equality of opportunities is assumed.

40.5. Example Applied to Universes of Subjects

Suppose we are dealing with a universe of subjects, from which a sample of **N** subjects has been drawn. In this case, the **theoretical probability** associated with each subject is equal to the inversion of N:

$$\text{Theoretical Probability per Subject} = \frac{1}{N}$$

This represents the statistical probability under the hypothesis of equal opportunities among the subjects in the sample, in the absence of bias. In other words, each subject is theoretically expected to have the same chance of occurrence or representation, independent of their empirical characteristics.

This abstraction is fundamental: it provides a **baseline measure** against which real-world deviations (biases) can be measured.

40.6. Example Applied to Annual Income of Individuals

Consider now the application of statistical probability to the distribution of **annual income among individuals**. Let the sample **N** represent the total number of individuals. The **direct score** of each individual is their annual income.

- The **empirical probability** of annual income for an individual is equal to their annual income divided by the total income of all individuals in the sample.
- The **theoretical probability**, under conditions of equality of opportunity, is simply the inversion of **N** (1/N). This represents the hypothetical probability that

each individual would have the same income if resources were perfectly distributed without bias.

Thus, theoretical probability in this context is not a measure of reality, but of *ideal equality*. It is a normative reference point, allowing us to assess how far empirical distributions of wealth diverge from the hypothetical baseline of equal opportunities.

This contrast between empirical and theoretical distributions becomes the foundation for calculating **Bias Levels**, which measure the degree to which reality deviates from the ideal of equality.

40.7. Difference Between Empirical and Theoretical Probability: Level of Bias

The difference between the empirical probability of annual income and the theoretical probability of annual income under conditions of equal opportunity is defined as the *Level of Bias*.

$$p(x_i) - \frac{1}{N} = \text{Level of Bias}$$

where:

$$\frac{1}{N} = \text{theoretical probability}$$
$$p(x_i) = \text{empirical probability}$$

40.8. Interpretation of Positive and Negative Bias

Individuals with higher annual income will have an empirical probability of income greater than the theoretical probability, and therefore their Level of Bias will be positive. Conversely, individuals whose empirical probability of income is lower than the theoretical probability will exhibit negative bias. The Level of Bias thus measures the deviation of each subject or option in the sample: every positive bias indicates that the

empirical probability is greater than the theoretical one, and every negative bias indicates that the empirical probability is lower than the theoretical one.

40.9. General Concept of Subject in Statistical Probability

In this context, the concept of *subject* does not refer strictly to a person or individual. Rather, it refers to any subject of a predicate, where the measurement of particular qualities yields a direct score. The empirical probability of the subject is then equal to its direct score divided by the sum of all direct scores in the sample.

40.10. Comparative Example Between Countries and Per Capita Income

Suppose we wish to conduct a comparative study of per capita income levels among a group of countries, whether in a specific region or continent, or on a global scale across all countries. For this purpose, the direct score of each country will be its average per capita income in a given year. The empirical probability is then defined as the ratio of the per capita income of each country to the total sum of the per capita incomes of all selected countries—whether regionally, continentally, or globally.

In this case, the sample *N* of subjects corresponds to the number of countries under study. The empirical probability of per capita income for each country is therefore equal to its per capita income divided by the sum of the per capita incomes of all selected countries. While the empirical probability of per capita income per country represents the real statistical probability of each country at the moment of measurement, the theoretical probability of per capita income per country is equal to dividing unity by the number of selected countries, $1/N$, whether on a regional, continental, or global scale.

40.11. Interpretation of Bias in International Studies

Whereas the empirical probability of per capita income per country provides an empirical measure of the real probability of each country based on its direct score, the

theoretical probability reflects the probability that all countries would have if they possessed the same per capita income under equal conditions. The difference between empirical probability and theoretical probability thus represents the bias of each country. Accordingly, countries with a higher average annual per capita income will show positive bias, while countries with lower average annual per capita income will show negative bias.

40.12. Application in Universes of Limited Options

In studies of limited options—those in which the distribution of the frequency of occurrence is analyzed over a given set of alternatives—the sample *N* corresponds to the number of options. In such cases, the empirical probability of each option is equal to its particular frequency divided by the total frequency of the entire sample. Thus, while the empirical probability measures the real probability of the occurrence of an option within a sample of *N* options, the theoretical probability measures the hypothetical probability that each option would have in the absence of bias, under conditions of equal opportunity.

40.13. Example of a Coin Toss

Suppose we toss a coin and study whether the outcome is heads or tails. There are only two possible options, so the sample *N* of options is limited to just two: heads or tails. The theoretical probability, in the absence of bias, is therefore 0.5, assuming that the coin is fair and that both alternatives are equally possible. While the theoretical probability is the same for all options, calculated as the reciprocal of *N*, $1/N$, the empirical probability of each option will be equal to its particular frequency divided by the total frequency.

40.14. Example of Labor Distribution by Gender

Consider an applied study in statistical probability on labor distribution by gender—whether one is male or female. In this case, $N = 2$, corresponding to the two options: male or female. The empirical probability of labor distribution for women is equal to the number of women employed divided by the total active population, while the empirical probability for men is equal to the number of men employed divided by the total active population.

Since the theoretical probability, the reciprocal of N, in this case is 0.5 (as there are only two options: male or female), the difference between empirical probability and theoretical probability represents the bias in labor distribution by gender. If we assume, on empirical grounds, that the probability of women participating in the labor force is lower than that of men, then women will exhibit a greater negative bias in labor distribution, while men will exhibit a greater positive bias.

40.15. Essential Difference Between Empirical and Theoretical Probability

While empirical probability represents the real statistical probability, theoretical probability represents the hypothetical statistical probability under conditions of equal opportunity or in the absence of bias. Theoretical probability is the probability that each subject or option should hypothetically possess if equality of opportunity were truly present, with no influencing factor other than chance itself.

40.16. Chance as Free Will in Equality of Opportunity

Insofar as chance represents the free will of what happens under conditions of equal opportunity, without any cause other than chance itself, the empirical distribution of labor between men and women should be the same, except for the random error generated by nature

.

40.17. General Definition of Theoretical Probability in Equality of Opportunity

Theoretical probability, in this sense, measures what the probability of each subject or option *should* theoretically be within a sample of *N* subjects or options, ***if all had exactly the same chances in the absence of bias. The difference between empirical and theoretical probability is the Level of Bias.*** Empirical probabilities greater than the theoretical value correspond to positive bias, whereas empirical probabilities lower than the theoretical value correspond to negative bias. Differential bias thus explains the differential behavior of subjects or options and serves as the basis for structuring empirical tendencies that deviate from the theoretical tendency.

40.18. Empirical Tendency versus Theoretical Tendency

While the theoretical tendency of behavior by chance should reflect equality of opportunity within the free will of randomness, empirical behavior reveals a tendency toward biased outcomes that deviate from the theoretical expectation. These deviations are what explain the particular differences of each subject or individual option in the sample.

40.19. Theoretical Probability as Hypothetical Probability by Chance

Theoretical probability, therefore, is the hypothetical statistical probability determined by chance, whereas empirical probability is the real statistical probability. The difference between them is the empirical bias of individual behavior.

40.20. Rational Critique of Equality of Opportunity

In studies where the ideal is equality of opportunity, a rational critique of this equality can be carried out from different perspectives using the Second Method proposed by *Impossible Probability*. Any rational critique must be applied both at the individual and sample levels, and it may take the form of intra-meditional or inter-meditional studies. At the individual, intra-meditional level, this includes the *Validity of Equality* or

Significance of Equality; at the sample level, it includes the *Critical Sample Level of Equality*. Equality can also be studied through critical proportions, as well as through inter-meditional studies.

40.21. Studies of Positive and Negative Bias

In studies of positive bias, for example, when among a set of subjects or options there exists one that, for whatever reason, is an ideal subject or option with increased empirical probability, the rational critique of the *Level of Bias* can be applied through *Validity of Positive Bias* or *Significance of Positive Bias*, and at the sample level through the *Critical Sample Level of Bias*. Similarly, in studies of negative bias, one can apply *Validity of Negative Bias* and *Significance of Negative Bias*.

40.22. Omega Models and Ideal Subsets

In addition to the general models of theoretical and empirical probability, *Impossible Probability* introduces the so-called **Omega Models**. These arise when, within a given sample **N**, there exists a subset of *ideal* subjects or options smaller than **N** yet larger than one. This subset constitutes the group of ideal cases that *may be analyzed separately from the totality of **N***.

The critical study of such subsets is carried out through the notion of **Omega Validity**. By focusing on the internal structure of these ideal subsets, Omega Models make it possible to measure how the presence of ideal elements modifies the distribution of probabilities and alters the balance between theoretical and empirical measures. This approach reflects one of the distinctive features of *Impossible Probability*: it not only examines universes as wholes but also incorporates the study of structured sub-universes within them.

40.23. Rational Critique Techniques in Impossible Probability

The Omega Models, along with many other forms of rational critique, are developed in detail in *Introduction to Impossible Probability*, particularly in section 11. Additional perspectives include:

- **Critical Proportions** (section 11), which evaluate how probability ratios deviate from theoretical expectations.
- **Direct Score Critiques** (section 12), which adapt rational criticism to direct measurements rather than proportions.
- **Typical Score Analysis** (section 15), which redefines the classical "standard score" within the framework of the Second Method.
- **Inter-meditional Critiques**, which analyze how different measurements or samples interact across distinct methodological contexts.

Together, these techniques create a comprehensive toolkit for critical rationalism applied to statistical probability, ensuring that both empirical data and theoretical constructs are subjected to rigorous logical evaluation.

40.24. The Inversion of N as Theoretical Probability and Arithmetic Mean

The inversion of **N** (1/N) serves multiple functions in Impossible Probability. Universally—whether in universes of subjects or universes of limited options—it functions as the **theoretical probability**. At the same time, it acts as the **arithmetic mean of empirical probabilities**, since the average of all empirical probabilities within a sample equals 1/N.

This dual role underscores the universality of inversion: it bridges the gap between hypothesis and observation. On the one hand, it sets the hypothetical baseline of equal opportunity; on the other, it quantifies the average of actual empirical distributions, thus unifying theoretical and empirical perspectives in a single operation.

40.25. Formula of the Inversion of N as Arithmetic Mean

Formally, the relationship can be expressed as:

$$\frac{\Sigma p(x_i)}{N} = \frac{1}{N}$$

That is, the arithmetic mean of empirical probabilities across a sample is always equal to the inversion of **N**. This formula reflects the structural harmony between theory and observation: regardless of individual variation, the overall average aligns with the baseline of equal distribution.

40.26. Specific Functions of Inversion of N in Universes of Subjects

Beyond its universal functions as theoretical probability and arithmetic mean, the inversion of **N** plays two additional essential roles specifically in **universes of subjects**:

1. **Theoretical probability of sample representativity error**.
2. **Theoretical probability of dispersion**.

Both of these functions are inversely proportional to the magnitude of the sample **N**. As **N** increases, representativity error decreases and dispersion diminishes; as **N** decreases, both error and dispersion increase. This makes 1/N a central regulator of statistical reliability in subject-based universes.

40.27. Inversion of N as Probability of Sample Representativity Error

The reason why inversion of **N** (1/N) measures the **probability of sample representativity error** is straightforward: in universes of subjects, the larger the sample size, the more representative the sample becomes of the universe. Consequently, the potential error of representativity decreases in inverse proportion to **N**.

This means that one of the primary methods of controlling sample error is simply to increase the size of **N**. An exception occurs when the entire population is taken as the sample. In such cases, representativity error reaches its natural limit, but even then, the population itself remains only a sample of its historical trajectory. Thus, *every*

population is itself a sample of its history, and representativity error is never entirely eliminated but rather relativized to temporal context.

40.28. Inversion of N as Probability of Theoretical Dispersion

Simultaneously, in universes of subjects, the inversion of **N** functions as the **probability of theoretical dispersion**. Under normal statistical conditions, as sample size increases, the direct scores tend to cluster around the arithmetic mean. Within Impossible Probability, the same principle is reformulated: as **N** increases, empirical probabilities tend to converge around 1/N.

Thus, inversion of **N** becomes both the arithmetic mean and the theoretical measure of dispersion. Larger **N** implies smaller dispersion; smaller **N** implies greater dispersion. This duality positions 1/N as a central metric for understanding how empirical distributions align with or deviate from theoretical baselines in universes of subjects.

40.29. The Multifunctional Value of Inversion of N

In conclusion, inversion of **N** (1/N) must be recognized as a **multifunctional constant** within Impossible Probability. Universally, across all types of universes, it serves as both the arithmetic mean and theoretical probability. Specifically, in universes of subjects, it also serves as the probability of representativity error and theoretical dispersion.

In all possible universes—whether finite or infinite, natural or social—the inversion of **N** expresses the same essential principle: the **theoretical probability of equal opportunity**. It is simultaneously a mathematical ratio, a methodological constant, and an epistemological foundation upon which the rational critique of empirical distributions is built.

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